Numerical Modeling of Hydromagnetic Squeeze Film in Conducting Longitudinally Rough Annular Plates

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ABSTRACT:

This paper aims to discuss the behavior of hydromagnetic squeeze film between longitudinally rough conducting annular plates. In view of the stochastic averaged process of Christensen and Tonder regarding roughness the associated Reynolds' type equation is derived by resorting to the usual equation of magnetohydrodynamic. The role of standard deviation come with the characteristic of roughness turns out to be contrary to that of transverse roughness. The negative effect of roughness can be countered suitably by the positive effect of hydromagnetization. In addition the load bearing capacity considerably increases with a suitable combination of conductivity and aspect ratio. A close glance at the results presented here suggests that, if proper designed than this type of bearings system may be favorable to the industry.

KEYWORDS: - Annular plates, Squeeze film, longitudinal roughness, hydromagnetic lubrication, load bearing capacity.

INTRODUCTION:

The theoretical study of magnetohydrodynamic pressure in liquid metal lubrication was introduced by Elco and Huges(1962). Magnetohydrodynamic squeeze film behavior was discussed by Kuzma (1964) and Kuzma et al. (1964). Shukla and Prasad (1965) have analyzed the behavior of hydromagnetic squeeze films between two conducting non-porous surfaces and studied the effect of the conductivities of surfaces on the performance of the bearing system. Patel and Hingu (1978) have dealt with magnetohydrodynamic lubrication. The study of hydromagnetic squeeze films between annular plates has been analyzed by Sinha and Gupta (1974), Patel and Gupta (1979) and Prajapati (1995).

In many theoretical studies of squeeze film Hydromagnetic Lubrication, deterministic models were considered where one surface was smooth and other was rough. In such cases the Reynolds equation was analyzed with the statistical averaging technique considering the surface roughness and the equation governing the fluid flow. After having some run-in and wear, the bearing surfaces are known to develop roughness. The effect of surface roughness has been analyzed by many investigators (Tzeng and Saibel (1967), Christensen and Tonder (1969a, 1969b, 1970), Berthe and Godet (1973)). For transverse as well as longitudinal surface roughness, Christensen and Tonder (1969a, 1969b, 1970) proposed a comprehensive general analysis. The approach of Christensen and Tonder (1969a, 1969b, 1970) was the basis of the analysis to discuss the effect of surface roughness in a number of investigations (Ting (1975), Guha (1993), Gupta and Deheri (1996), Andharia et al. (1997, 1999), Patel and Deheri (2004), Andharia and Deheri (2010, 2013), Shimpi and Deheri (2016), Lin (2016) and Adeshara et. al. (2018)).

However, as indicated by Andharia and Deheri (2010, 2013), Andharia et. Al. (1997), Shimpi and Deheri (2016), Lin (2016) and Adeshara et. al. (2018), the longitudinal roughness pattern was a bit sober. All these above observations make it clear that from the longevity point of view of bearing, the effect of standard deviation remains very much crucial.

Vadher et. al. (2008) considered hydromagnetic squeeze films between conducting rough porous annular plates. Therefore, in the present article it has been proposed to study the effect of longitudinal rough pattern on the above configuration of the bearing system.

ANALYSIS:

Figure: I indicate the configuration of the bearing system.

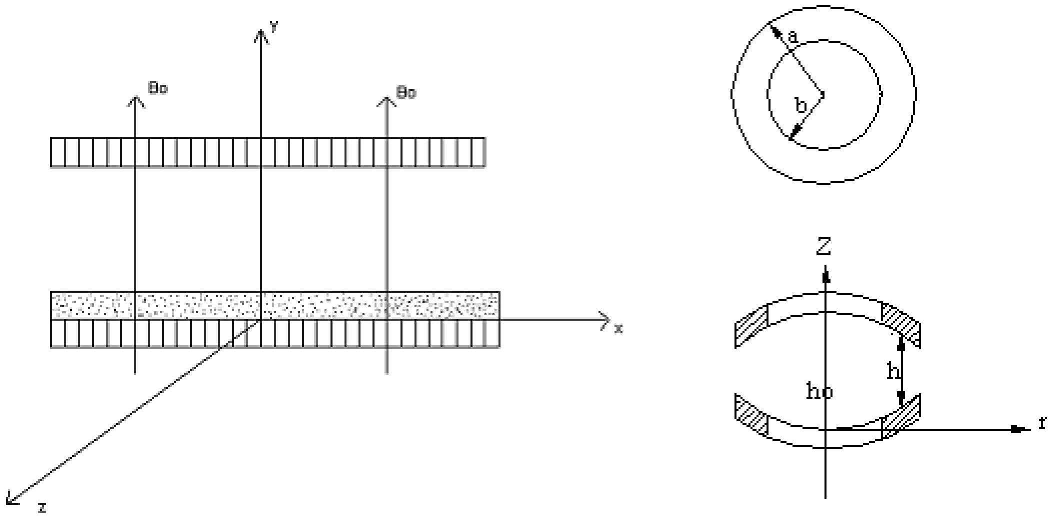

Figure: I Configuration of the bearing system.

It is assumed that the lower plate is fixed while the upper plate moves towards the lower plate along its normal. The annular plates are considered to be electrically conductive and a lubricant that conducts electrically fills the clearance space between them. A uniform transverse magnetic field is applied between the annular plates. The modified form of Darcy's law (Prajapati (1995)) is adopted for flow in the porous medium. The film region's equations of hydromagnetic lubrication theory hold good.

Under usual assumptions of hydromagnetic lubrication, the modified Reynolds' equation for the lubricant film pressure is (Prajapati (1995), Patel and Deheri (2004), Vadher et. al. (2008)) is obtained in the following form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\dot{h}m(h)}{\frac{2}{M^3}\left(\frac{M}{2}-\tanh\left(\frac{M}{2}\right)\right)\left[\frac{\phi_0+\phi_1+1}{\phi_0+\phi_1+\tanh\frac{M}{2}/\frac{M}{2}}\right]} \qquad \dots\dots\dots (1)$$

where

$$m(h) = h^{-3}[1-\alpha h^{-1}+6h^{-2}(\sigma^2+\alpha^2)-10h^{-3}(\varepsilon+3\sigma^2\alpha+\alpha^3)$$

Solving this equation by making use of boundary conditions

$$p(a) = 0;\ p(b) = 0 \qquad \dots\dots\dots (2)$$

one gets the pressure distribution as

$$p = \frac{\mu\dot{h}\, m(h)(a^2-b^2)\left[\frac{\ln\left(\frac{r}{b}\right)}{\ln\left(\frac{a}{b}\right)}-\frac{\left(\frac{r}{b}\right)^2-1}{\left(\frac{a}{b}\right)^2-1}\right]}{\frac{8}{M^3}\left(\frac{M}{2}-\tanh\left(\frac{M}{2}\right)\right)\left[\frac{\phi_0+\phi_1+1}{\phi_0+\phi_1+\tanh\frac{M}{2}/\frac{M}{2}}\right]}$$

The pressure distribution in non-dimensional form is obtained as

$$P = -\frac{h^3 p}{\mu\dot{h}\pi(a^2-b^2)}$$

$$P = -\frac{h^3 p}{\mu\dot{h}\pi(a^2-b^2)} = \frac{M(h)\left[\frac{\ln\left(\frac{r}{b}\right)}{\ln\left(\frac{a}{b}\right)}-\frac{\left(\frac{r}{b}\right)^2-1}{\left(\frac{a}{b}\right)^2-1}\right]}{\frac{8}{M^3}\left(\frac{M}{2}-\tanh\left(\frac{M}{2}\right)\right)\left[\frac{\phi_0+\phi_1+1}{\phi_0+\phi_1+\tanh\frac{M}{2}/\frac{M}{2}}\right]} \qquad \dots\dots\dots (3)$$

where

$\sigma^* = (\sigma/h)$, $\alpha^* = (\alpha/h)$, $\varepsilon^* = (\varepsilon/h^3)$

and

$$M(h) = 1-3\alpha^*+6\left(\sigma^{*2}+\alpha^{*2}\right)-10(\varepsilon^*+3\sigma^{*2}\alpha^*+\alpha^{*3})$$

Then the load carrying capacity given by

$$w = 2\pi\int_a^b p(r).r\,dr$$

is calculated in non-dimensional form as

$$W = -\frac{wh^3}{\mu\dot{h}\pi^2(a^2-b^2)}$$

$$W = \frac{M(h)\left[\frac{\left(\frac{a}{b}\right)^2+1}{\left(\frac{a}{b}\right)^2-1}-\frac{1}{\log\left(\frac{a}{b}\right)}\right]}{\frac{8\pi}{M^3}\left(\frac{M}{2}-\tanh\left(\frac{M}{2}\right)\right)\left[\frac{\phi_0+\phi_1+1}{\phi_0+\phi_1+\tanh\frac{M}{2}/\frac{M}{2}}\right]} \quad \ldots\ldots\ldots (4)$$

RESULTS AND DISCUSSIONS:

It is clear from equations (3) and (4) that the pressure distribution and the load bearing capacity depend on various parameters such as: M, $\phi_0 + \phi_1$, k, σ*, ε* and α*. Further, it is noticed that as M increases W increases for fixed values of $\phi_0 + \phi_1$, k, σ*, α* and ε*. Besides, the effect of conductivity on the load distribution W comes through the factor

$$\left(\frac{\phi_0+\phi_1+\frac{\tanh(M/2)}{(M/2)}}{\phi_0+\phi_1+1}\right)$$

For large values of M, this tends to

$$\frac{\phi_0+\phi_1}{\phi_0+\phi_1+1}$$

as $\tanh M \approx 1$, $\frac{2}{M} \approx 0$. It may be observed from the mathematical analysis also that as $\phi_0 + \phi_1$ increases the pressure, load carrying capacity. It is also perceived that the bearing with hydromagnetic field can support a load even when there is no flow.

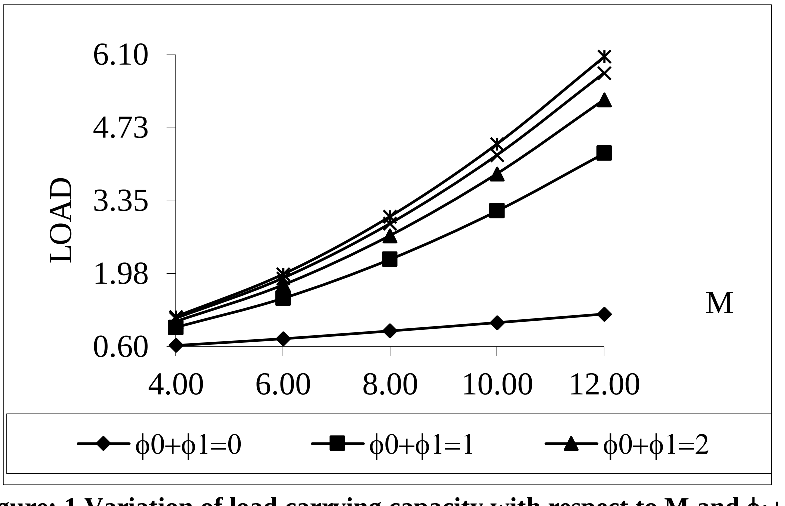

Figure: 1 Variation of load carrying capacity with respect to M and $\phi_0+\phi_1$

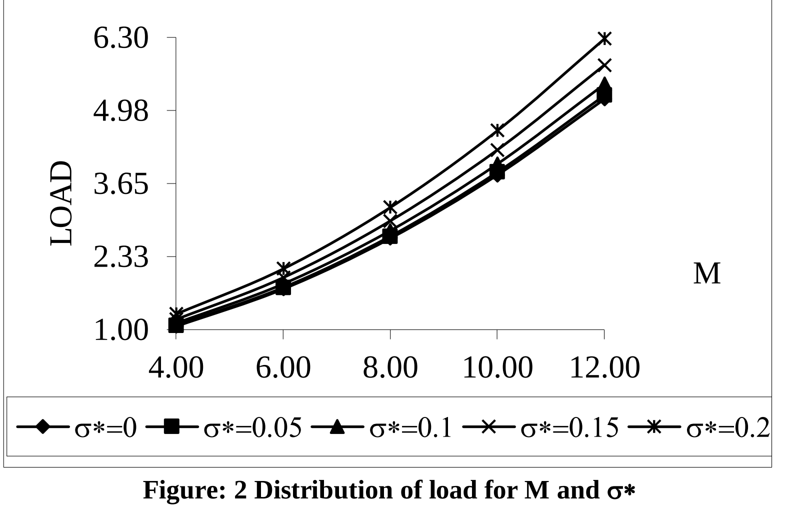

Figure: 2 Distribution of load for M and σ*

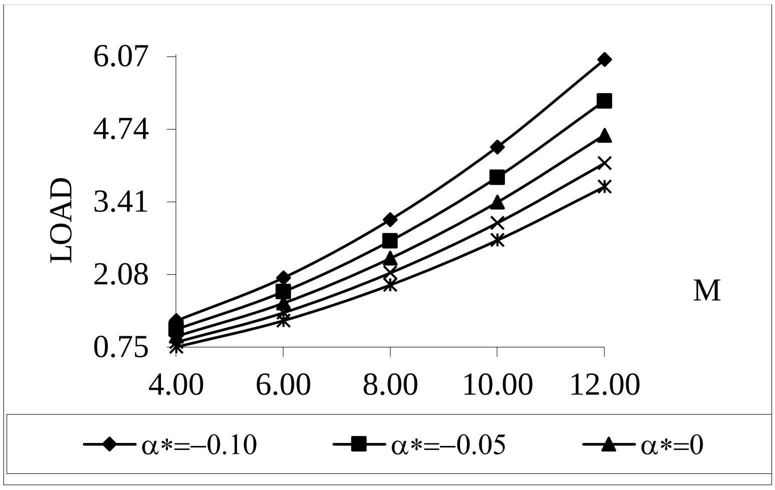

Figure: 3 Profile of load bearing capacity with regards to M and α*

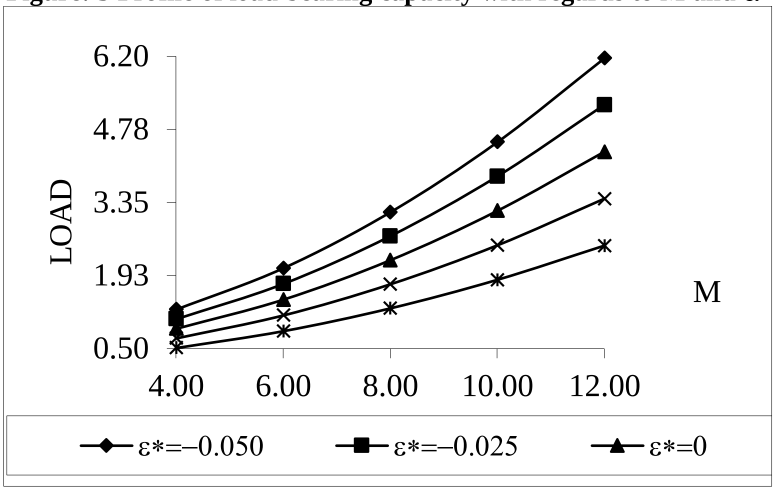

Figure: 4 Variation of load carrying capacity with respect to M and ε*

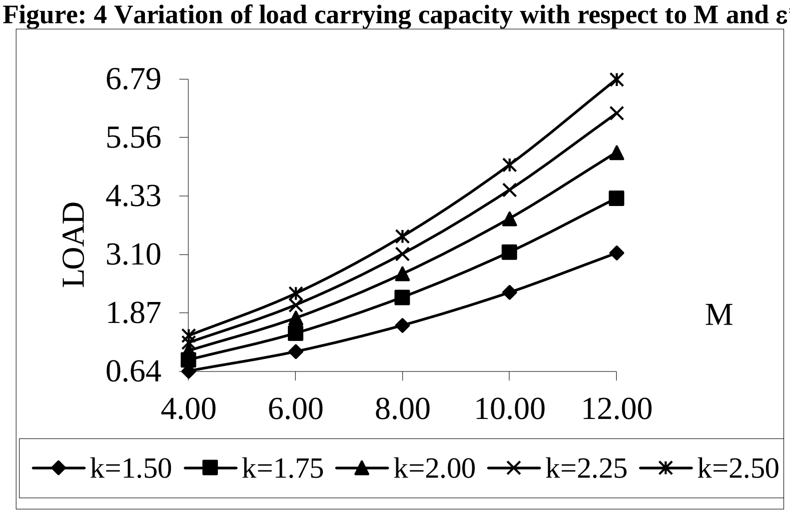

Figure: 5 Variation of load carrying capacity with respect to M and k

Figures (1) to (5) depict the profile of load carrying capacity with respect to the hydromagnetization parameter M for different values of conductivity parameter $\phi_0 + \phi_1$, standard deviation σ*, variance α*, measure of symmetry ε* and the aspect ratio k respectively. All these figures clearly mention that the load carrying capacity increases with the increasing values of the magnetization parameter M. One can find the distribution of load carrying capacity with respect to conductivity parameter in Figures (6) to (9). It is evident that the load carrying capacity increases significantly with respect to the conductivity. Here, the effect of negative variance is also quite significant in increasing the load carrying capacity.

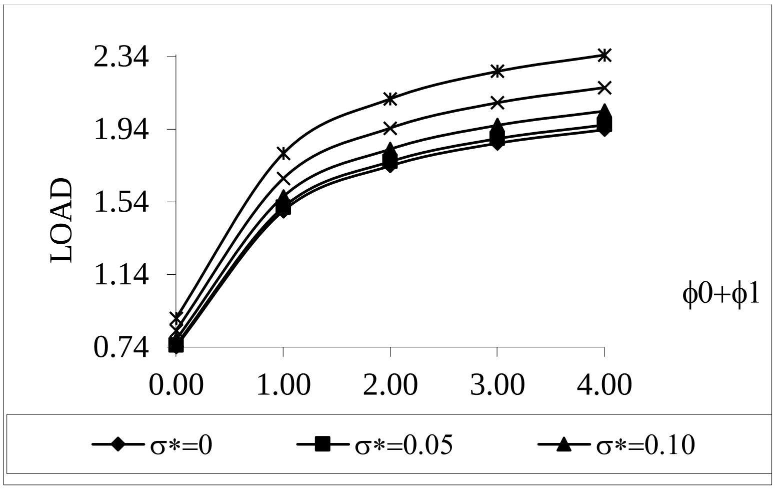

Figure: 6 Variation of load carrying capacity with respect to $\phi_0+\phi_1$ and σ*

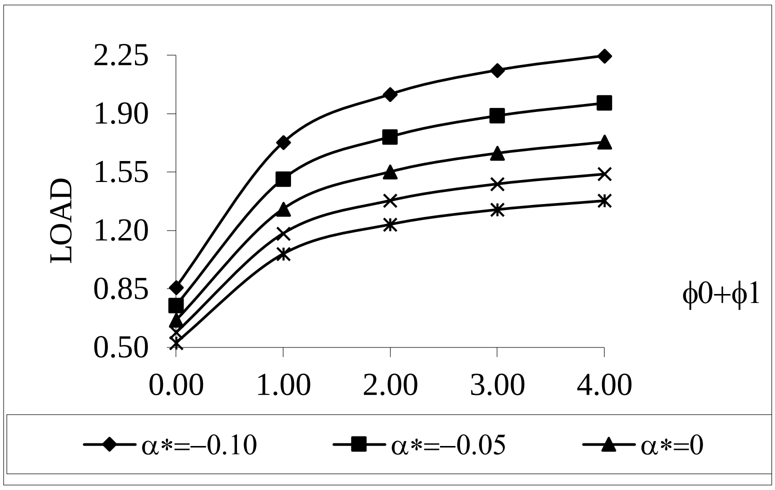

Figure: 7 Distribution of load for $\phi_0+\phi_1$and α*

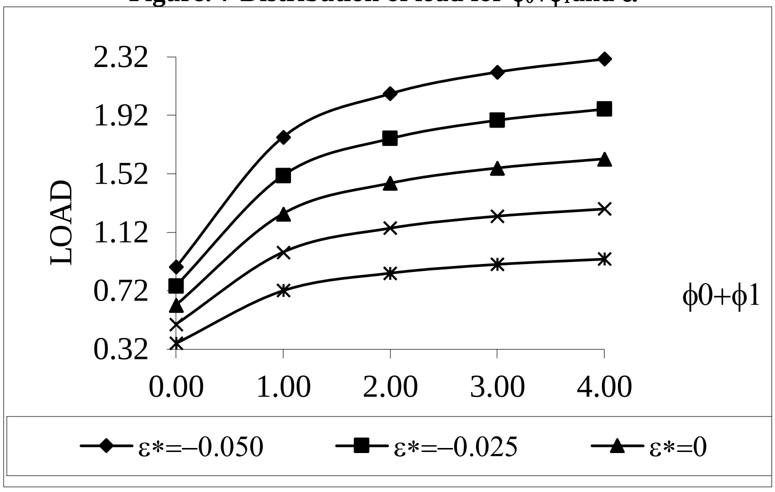

Figure: 8 Profile of load bearing capacity with regards to $\phi_0+\phi_1$ and ε*

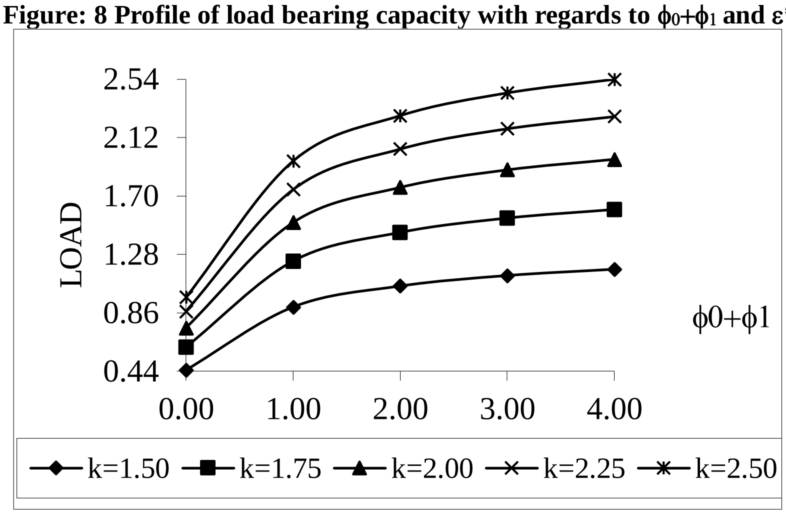

Figure: 9 Profile of load bearing capacity with regards to $\phi_0+\phi_1$ and k

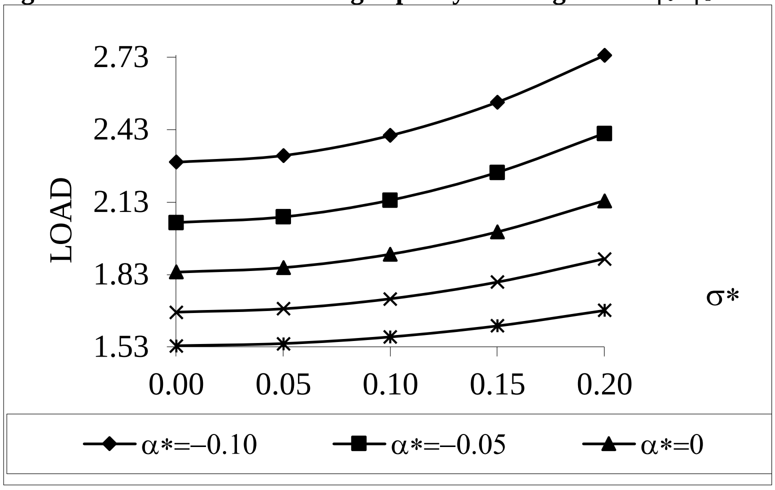

Figure: 10 Profile of load bearing capacity with regards to σ* and α*

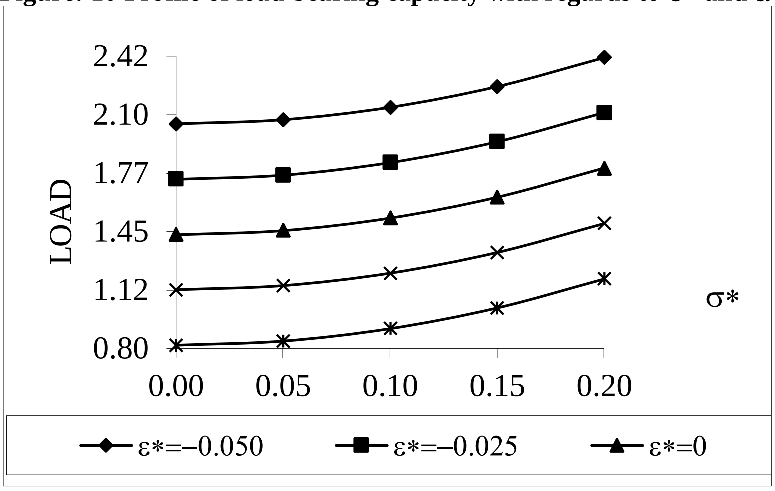

Figure: 11 Variation of load carrying capacity with respect to σ* and ε*

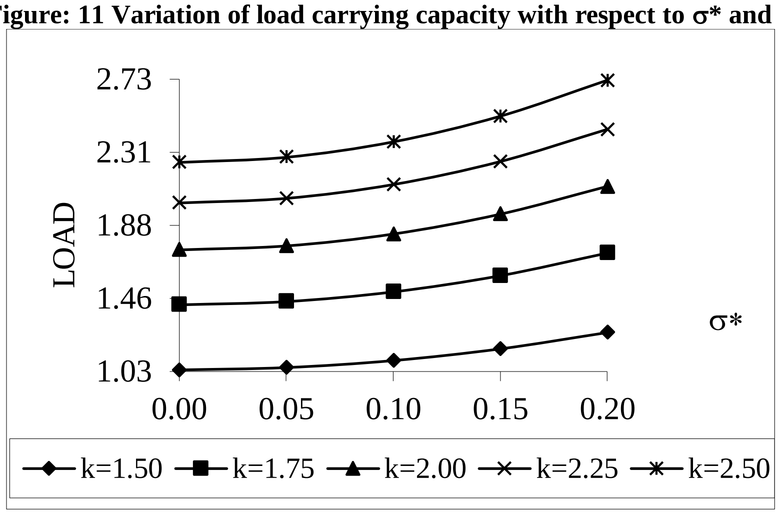

Figure: 12 Variation of load carrying capacity with respect to σ* and k

The net effect of standard deviation associated with roughness is presented in Figures (10) to (12). It is seen that standard deviation enhances the load profile, which is totally opposite as compare to the transverse roughness. Also, the effect of variance and skewness is depicted from Figures (13 – 14) and Figure (15) respectively. Here, the negative variance and skewness increases the load carrying capacity.

These Figures show that roughness, in general, adversely affects the bearing system. Besides, increasing values of aspect ratio cause increased load (c.f. Figure: 15). It is revealed that the effect of standard deviation is very much positive for performance point of view.

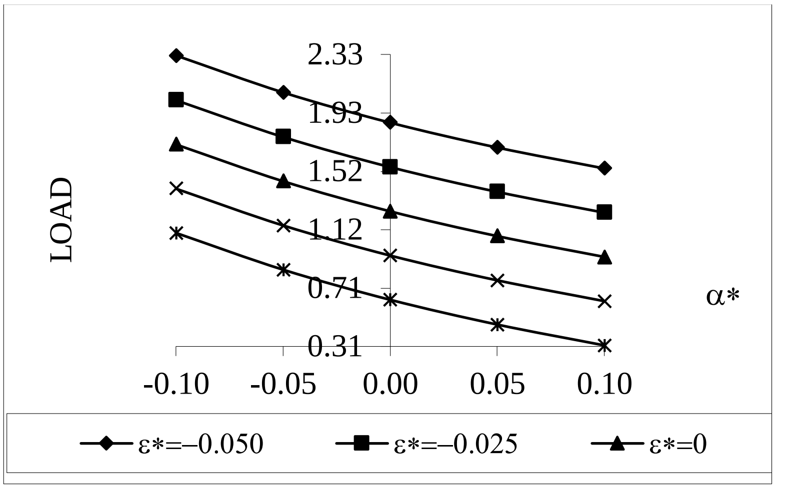

Figure: 13 Variation of load carrying capacity with respect to α* and ε*

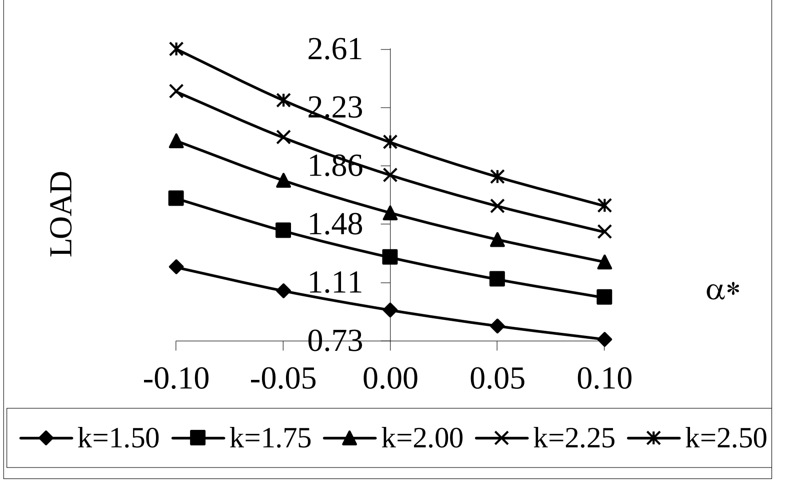

Figure: 14 Profile of load bearing capacity with regards to α* and k

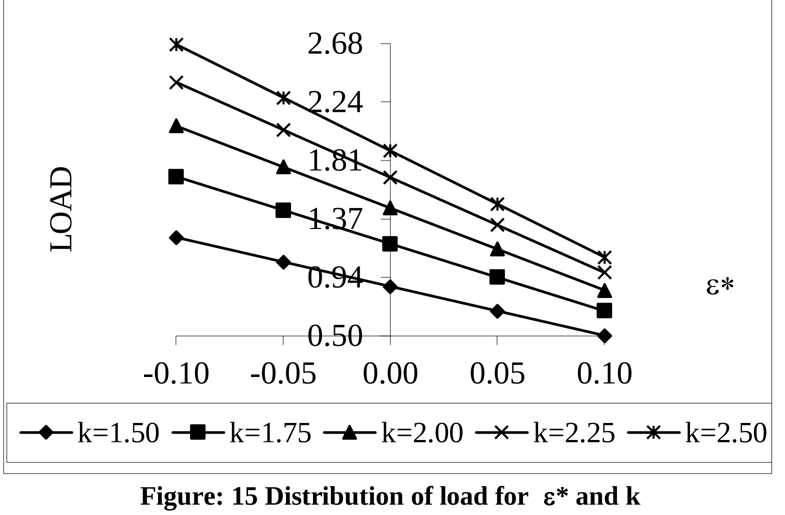

Figure: 15 Distribution of load for ε* and k

This investigation establishes that the negative effect induced by variance (+ve), positive skewness can be compensated up to a large extent by suitably choosing the hydromagnetization parameter M, conductivity $\phi_0 + \phi_1$, standard deviation σ* and the aspect ratio k in the case of negatively skewed roughness, especially when negative variance is involved. Thus, it is suggested that longitudinal roughness must be given due consideration while designing the bearing system. The analysis incorporated here modifies and develops the earlier analysis concerning the hydromagnetic squeeze film in annular plates and presents at least an additional degree of freedom to compensate the adverse effect.

CONCLUSION:

The negative effect of a few roughness parameters can be remunerated up to a certain extent by suitably choosing the plate conductivities, the magnetization parameter, the aspect ratio and standard deviation in the case of negatively skewed roughness. This compensation gets further improved especially when negative variance is involved. Furthermore, this study offers ample scopes for the extension of the life period of the bearing system by observing that the rough bearing with hydromagnetic fluid can support a load even when there is no flow. Hence, it can be concluded that the bearing system registered an improved performance owing to hydromagnetization and standard deviation associated with longitudinal roughness.

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