A study on the performance of longitudinally rough hydromagnetic squeeze film between conducting infinitely long rectangular plates

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Abstract—**An effort has been made to discuss the effect of longitudinal roughness on the performance of a hydromagnetic squeeze film between conducting infinitely long rectangular plates with electrically conducting lubricant in the presence of a transverse magnetic field. Here, the bearing surfaces are taken to be longitudinally rough. The random roughness is categorized by a stochastic random variable with mean, variance and skewness. The related Reynolds' equation is stochastically averaged with respect to the random roughness parameter. This equation is resolved with proper boundary conditions to obtain the pressure distribution, which is used to calculate the load bearing capacity. The results are presented graphically.**

Keywords— **Hydromagnetic lubrication, squeeze film, Reynolds' type equation, longitudinal roughness, load carrying capacity**

Nomenclature:

r	**Radial coordinate**
a	**Semi-major axis**
b	**Semi-minor axis**
k	**Aspect ratio (a/b)**
h	**Lubricant film thickness**
B0	**Uniform transverse magnetic field applied between the plates.**
h0	**Initial film thickness**
s	**Electrical conductivity of the lubricant**
μ	**Viscosity**
M	**Hartmann number**
h'_0	**Surface width of the lower plate**
h'_1	**Surface width of the upper plate**
S_0	**Electrical conductivity of lower surface**
s_1	**Electrical conductivity of upper surface**
ϕ0(h)	**= = Electrical permeability of the lower surface**
ϕ1(h)	**= = Electrical permeability of the upper surface**
p	**Lubricant pressure**
w	**Load carrying capacity**
σ*	**Non-dimensional standard deviation (σ/h)**
α*	**Non-dimensional variance (α/h)**

ε* **Non-dimensional skewness (ε/h3)**
P **Non-dimensional pressure**
W **Dimensionless load carrying capacity**

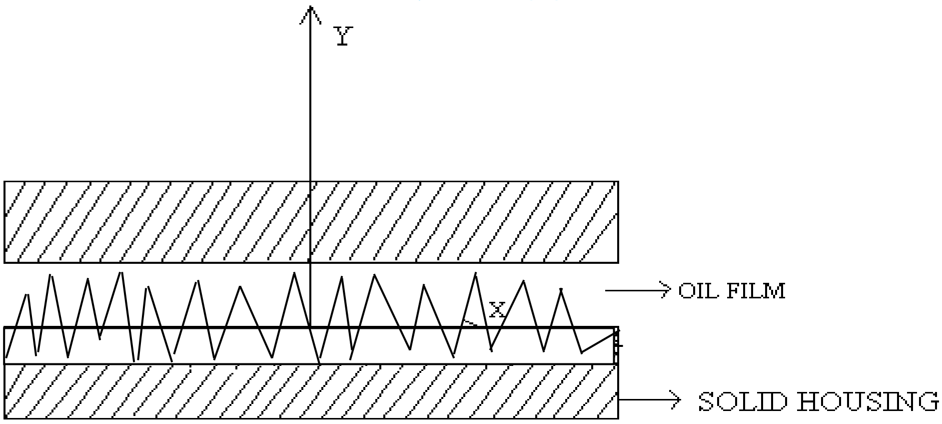
I. Introduction

Wu (1972.a, 1972.b) and Prakash and Vij (1973) analyzed and discussed the behaviour of the squeeze film when one surface was porous and backed by an impermeable solid wall Prakash and Vij considered a number of geometries, including circular, annular, elliptical and conical. Also included in this article was the squeeze film performance between infinitely long rectangular plates.Patel and Gupta (1979) discussed the effect of a transverse magnetic field on squeeze film behavior between porous plates of various geometries. Shukla (1965) and Kuzma (1964) analyzed the behavior of hydromagnetic squeeze film between two non - porous conducting surfaces and investigated the effect of surface conductivity on load carrying capacity and approach time. They considered two geometric forms: an infinitely long rectangular and parabolically curved circular shape. It was observed that the load carrying capacity decreased when the bearing surfaces were non - conductive compared to the corresponding hydromagnetic case. However, it was possible to increase the load carrying capacity and approach time by increasing the surface conductivity. Prajapati (1995) discussed hydromagnetic squeeze film between different geometry porous plates.The bearing surfaces tend to be rough due to elastic deformation and wear. In particular, bearing surfaces develop roughness after some run in and wear. Many investigators have analyzed the effect of surface roughness (Davies (1963); Burton (1963); Michell (1950), Tonder (1972); Tzeng and Saibel (1967); Christensen and Tonder (1969.a, 1969b; 1970); Berthe and Godet (1973)). For both transverse and longitudinal surface roughness, Christensen and Tonder (1969a; 1969b; 1970) proposed a comprehensive general analysis. The approach of Christensen and Tonder formed the basis for the study of the effect of surface roughness in several investigations (Ting (1975), Prakash and Tiwari (1982); Prajapati (1991; 1992); Guha (1993); Gupta and Deheri (1996) ; Andharia, Gupta and Deheri (1997 ; 1999)).

The longitudinal roughness pattern was subjected to investigation in Andharia and Deheri (2001, 2010, and 2013). Transverse roughness pattern in the presence of a magnetic fluid has been the matter of investigation in Lin et. al. (2013))

The effect of transverse surface roughness on the performance of a hydromagnetic squeeze film between conducting infinitely long porous rectangular plates is being studied and analyzed.

II. ANALYSIS



The geometry and configuration of the bearing system is shown below

The lower plate with is considered to be fixed while upper plate moves along its normal towards the lower plate. The rectangular plates are taken to be electrically conducting and the clearance space between them is filled by an electrically conducting lubricant. A uniform transverse magnetic field is applied between the rectangular plates. The flow in the film region the equations of hydromagnetic lubrication theory hold.

When a large external electromagnetic field through the electrically conducting lubricant is applied, it gives rise to induced circulating currents, which in turn, interacts with the magnetic field and creates a body force called Lorentz force. This extra electromagnetic pressurization pumps the fluid between the bearing surfaces. In such a case Navier-Stokes equation for a steady, incompressible and isoviscous liquid gets modified as

$$\rho\left(\vec{q}\bullet\nabla\right)\vec{q} = -\nabla p + \mu\nabla^2\vec{q} + \vec{J}\times\vec{B}_0 \tag{1}$$

$$\nabla\bullet\vec{q} = 0 \tag{2}$$

The Maxwell’s equations and the Ohm’s law governing electromagnetic phenomena are:

$$\nabla\bullet\vec{B} = 0 \tag{3}$$

$$\nabla \times \vec{B} = \mu_e \vec{J} \tag{4}$$

$$\nabla \bullet \vec{E} = 0 \tag{5}$$

$$\nabla \times \vec{E} = 0 \tag{6}$$

and

$$\vec{J} = \sigma\left(\vec{E} + \vec{q} \times \vec{B}_0\right) \tag{7}$$

The equations of motion in the x- and z- directions are obtained from the equations (1) and (3) – (6), take the form

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - J_z B_0 = 0 \tag{8}$$

$$-\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial y^2} + J_x B_0 = 0 \tag{9}$$

and the electric currents in these directions from the equation (A_7) are obtained as

$$J_x = \sigma\left(E_x - wB_0\right) \tag{10}$$

$$J_z = \sigma\left(E_z + uB_0\right) \tag{11}$$

Substituting these values of J_x and J_z in equations (A_8) and (A_9), we get,

$$\frac{\partial^2 u}{\partial y^2} - \frac{M^2}{h^2} u = \frac{1}{\mu}\frac{\partial p}{\partial x} + \frac{M}{h}\sqrt{\frac{\sigma}{\mu}} E_z \tag{12}$$

$$\frac{\partial^2 v}{\partial y^2} - \frac{M^2}{h^2} v = \frac{1}{\mu}\frac{\partial p}{\partial z} + \frac{M}{h}\sqrt{\frac{\sigma}{\mu}} E_x \tag{13}$$

where

$$M = B_0 h \left(\frac{\sigma}{\mu}\right)^{1/2}$$ = Hartman number.

and

$$\frac{\partial p}{\partial y} = 0 \tag{14}$$

Solving equations (12) and (13) with necessary boundary conditions we obtain the values of u and v given by

$$u = \frac{h^2}{\mu M^2}\frac{\partial p}{\partial x}\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{M/2}}\right]\left[\frac{\cosh\frac{M}{2}\left(\frac{2y}{h} - 1\right)}{\cosh\frac{M}{2}} - 1\right] \tag{15}$$

$$v = \frac{h^2}{\mu M^2}\frac{\partial p}{\partial z}\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{M/2}}\right]\left[\frac{\cosh\frac{M}{2}\left(\frac{2y}{h} - 1\right)}{\cosh\frac{M}{2}} - 1\right] \tag{16}$$

The velocity of the lubricant in the porous region satisfies the modified Darcy's law, equation of continuity and generalized Ohm's law.

In the present case, we have, for the porous region:

$$\bar{u} = \left[-\frac{K}{\mu}\frac{\partial p}{\partial x} - K\sqrt{\frac{\sigma}{\mu}}\frac{M}{h}E_z\right]\frac{1}{C^2} \tag{17}$$

$$\overline{w} = -\frac{K}{\mu}\frac{\partial p}{\partial y} \tag{18}$$

$$\overline{v} = \left[-\frac{K}{\mu}\frac{\partial p}{\partial z} - K\sqrt{\frac{\sigma}{\mu}}\frac{M}{h}E_x\right]\frac{1}{C^2} \tag{19}$$

where

$$C^2 = 1 + \frac{K}{m}\frac{M^2}{h^2}$$

Using equations (15 - 19) in the equation of continuity and simplifying it one gets the modified Reynolds' equation as

$$\nabla^2 p = \frac{dh/dt}{\left[\frac{2h^3}{\mu M^3}\left\{\left(\tanh\frac{M}{2} - \frac{M}{2}\right)\right\}\right]\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{M/2}}\right]} \tag{20}$$

$$\frac{\partial^2 p}{\partial z^2} = \frac{\dot{h}\,\mu A}{\left[\frac{2}{M^3}\left(\tanh\frac{M}{2} - \frac{M}{2}\right)\right]} \bullet \frac{1}{\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}}\right]} \tag{21}$$

where

$$A = h^{-3}[1 - \alpha h^{-1} + 6h^{-2}(\sigma^2 + \alpha^2) - 10h^{-3}(\varepsilon + 3\sigma^2\alpha + \alpha^3)]$$

Solving this equation with the concerned boundary conditions,

$$p\left(\pm\frac{b}{2}\right) = 0 \tag{22}$$

leads to the distribution of pressure as

$$p = \frac{-\dot{h}\,b^2 \cdot \left(\frac{1}{4} - \frac{z^2}{b^2}\right)B}{2\left[\frac{2}{M^3}\left(\tanh\frac{M}{2} - \frac{M}{2}\right)\right]} \bullet \frac{1}{\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}}\right]}$$

The dimensionless pressure distribution is obtained as

$$P = \frac{-h^3 p}{\mu \dot{h}\,ab}$$

$$= \frac{\frac{1}{2(a/b)} \cdot \left(\frac{1}{4} - \frac{z^2}{b^2}\right)B}{\left[\frac{2}{M^3}\left(\tanh\frac{M}{2} - \frac{M}{2}\right)\right]} \bullet \frac{1}{\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}}\right]} \tag{23}$$

where in

$$B = 1-3\alpha^*+6(\sigma^{*2}+\alpha^{*2})-10(\varepsilon^*+3\sigma^{*2}\alpha^*+\alpha^{*3})$$

Then the load carrying capacity given by

$$w = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} p\cdot dxdz$$

$$= \frac{-\dot{h}\, b^3 a\, B}{12\left[\frac{2}{M^3}\left(\tanh\frac{M}{2}-\frac{M}{2}\right)\right]} \bullet \frac{1}{\left[\dfrac{\phi_0+\phi_1+1}{\phi_0+\phi_1+\dfrac{\tanh(M/2)}{(M/2)}}\right]}$$

The load carrying capacity in dimensionless form is obtained as

$$W = -\frac{wh^3}{\mu \dot{h}\, a^2 b^2}$$

$$= \frac{\dfrac{B}{12(a/b)}}{\left[\frac{2}{M^3}\left(\tanh\frac{M}{2}-\frac{M}{2}\right)\right]} \bullet \frac{1}{\left[\dfrac{\phi_0+\phi_1+1}{\phi_0+\phi_1+\dfrac{\tanh(M/2)}{(M/2)}}\right]} \tag{24}$$

III. RESULTS AND DISCUSSIONS

One can see that for smooth bearing surfaces this investigation reduces to the study of Prajapati (1995). Further, when conductivities are taken to be zero the investigation of Prakash and Vij (1973), can be obtained by considering $M \to 0$.

The factor

$$\left(\frac{\phi_0+\phi_1+\dfrac{\tanh(M/2)}{(M/2)}}{\phi_0+\phi_1}\right)$$

is responsible for the effect of conductivity on W. However, as $\tanh(M) \cong 1$ and $(2/M) \cong 0$ for large values M, the above factor tends to

$$\frac{\phi_0+\phi_1}{\phi_0+\phi_1+1}.$$

So it can be easily seen that the LCC gets maximize as the values of parameter of conductivity ($\phi_0+\phi_1$) increases (Figs. (6) to (9)). A closed scrutiny of equation (W) indicates that in the absense of flow the bearing with the magnetic fluid can support a certain amount of load.

Variation of LCC with respect to the magnetization parameter presented in Figs. (1) to (5) suggest that the LCC increases as the magnetization parameters increases. A noticable difference in the role of standard deviation is found here as the load increases with increase in standard deviation (Figs. (10) to (12). However, the trends of LBC with resoect to skewness and variance remain identical with the case of transverse roughness. Which can be seen from Figs. (13) to (15).

It is also found that influence of aspect ratio is registered to be equally strong. Besides, it is appealing to note that the effect of skewness remains almost marginal in most of the cases (Figs. (4), (8), (11)). It was observed that the performance of the bearing remains a little better when the plates are considered to be non - conductive.

A comparison of the present analysis with that of makes it clear that comparision of negative effect introduced by roughness is comperatively more.

IV Conclusions

It is easily observed that the longitudinal roughness remains more favorable as compared to the transverse roughness case. In fact, the standard deviation makes all the difference. Hence the roughness needs to be considered carefully while designing the bearing system. A close observation indicates that some amount of load is supported by the system even if no flow occurs. This does not happen in the case of traditional fluid based bearing system.

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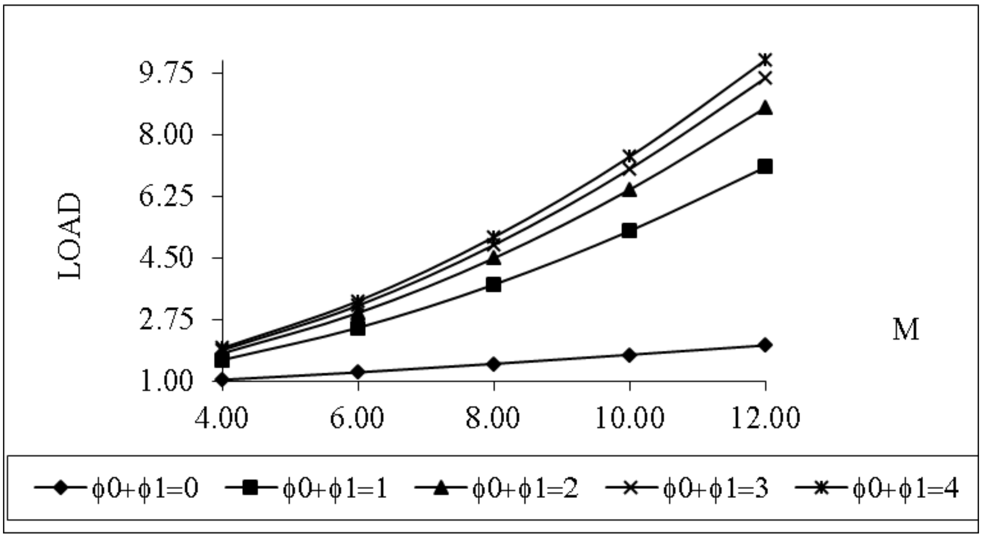

Figure 1: Variation of load carrying capacity with respect to M and $\phi_0+\phi_1$

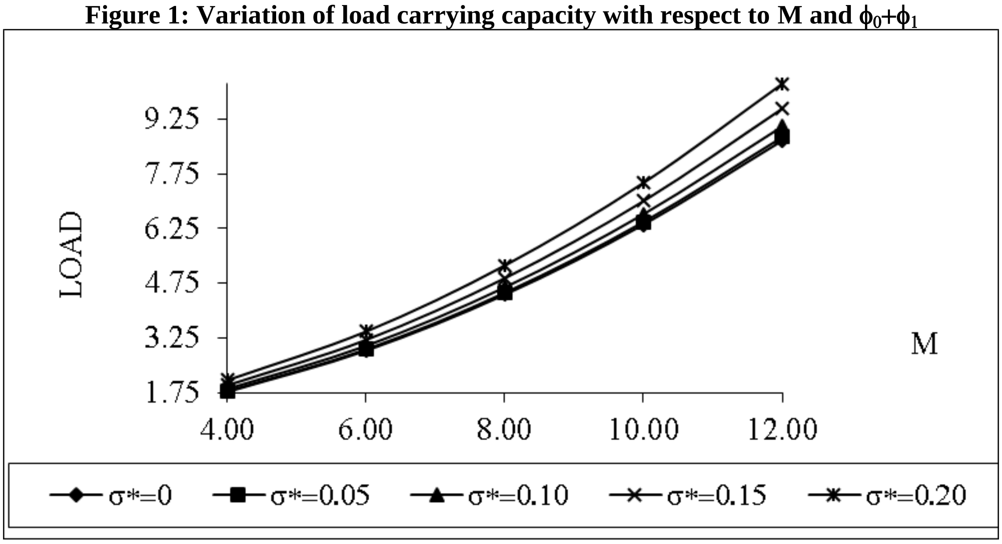

Figure 2: Distribution of load with respect to M and σ*

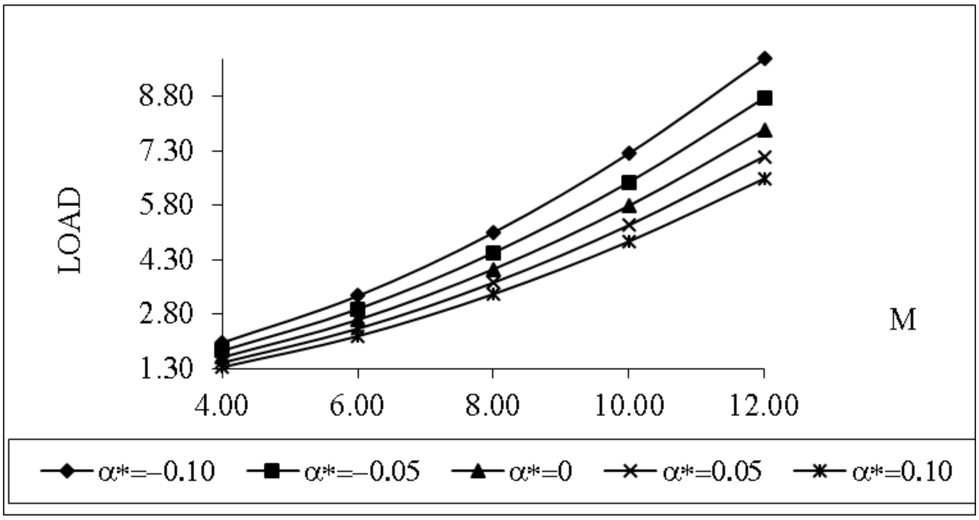

Figure 3: Profile of load bearing capacity with regards to M and α*

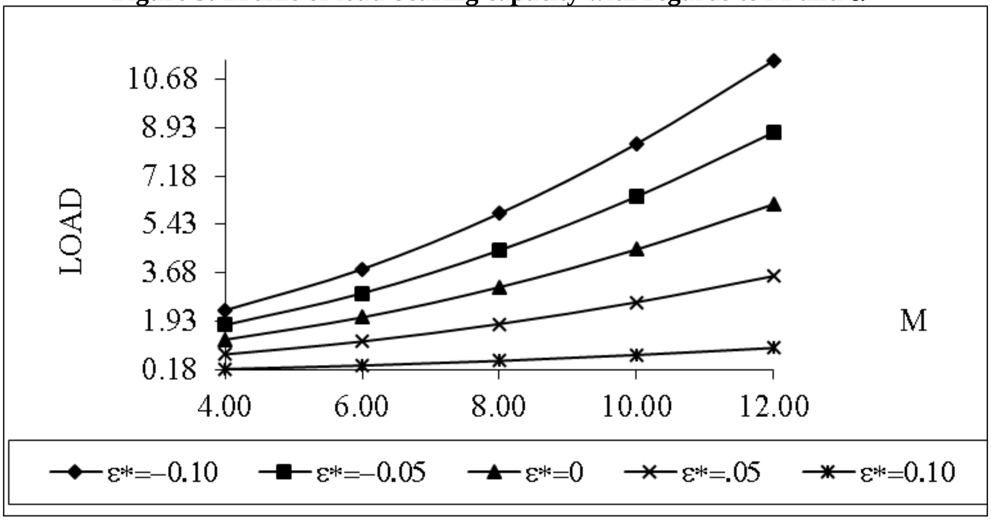

Figure 4: Variation of load carrying capacity with respect to M and ε*

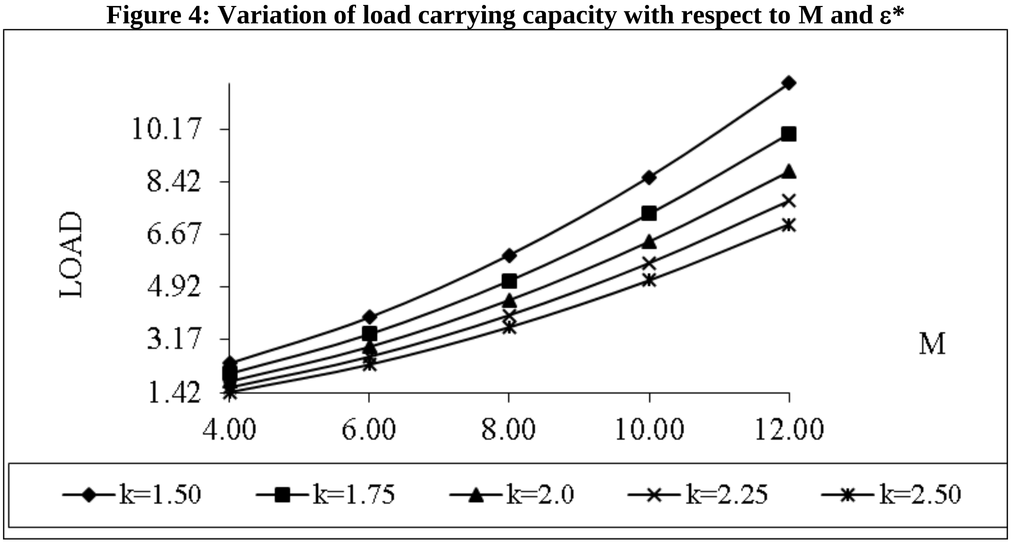

Figure 5: Distribution of load with respect to M and k

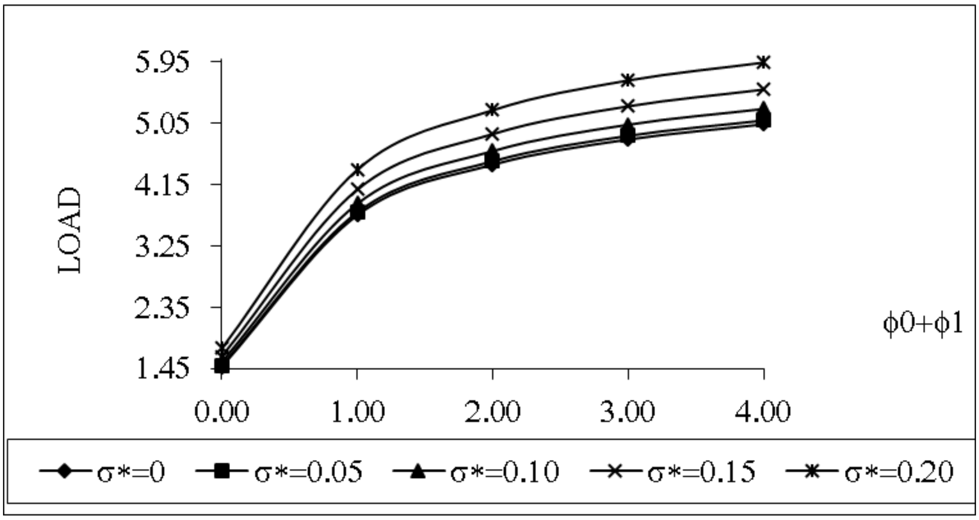

Figure 6: Profile of load bearing capacity with regards to $\phi_0+\phi_1$ and σ*

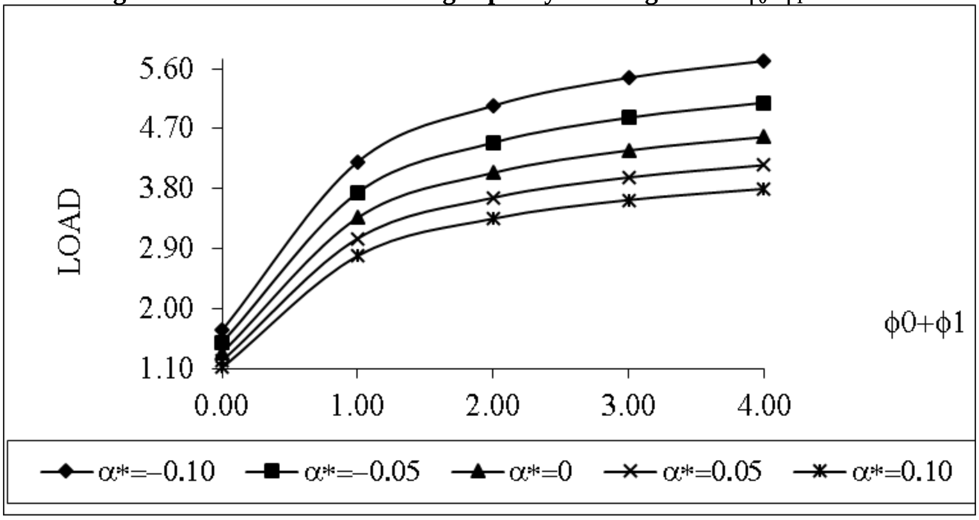

Figure 7: Variation of load carrying capacity with respect to $\phi_0+\phi_1$ and α*

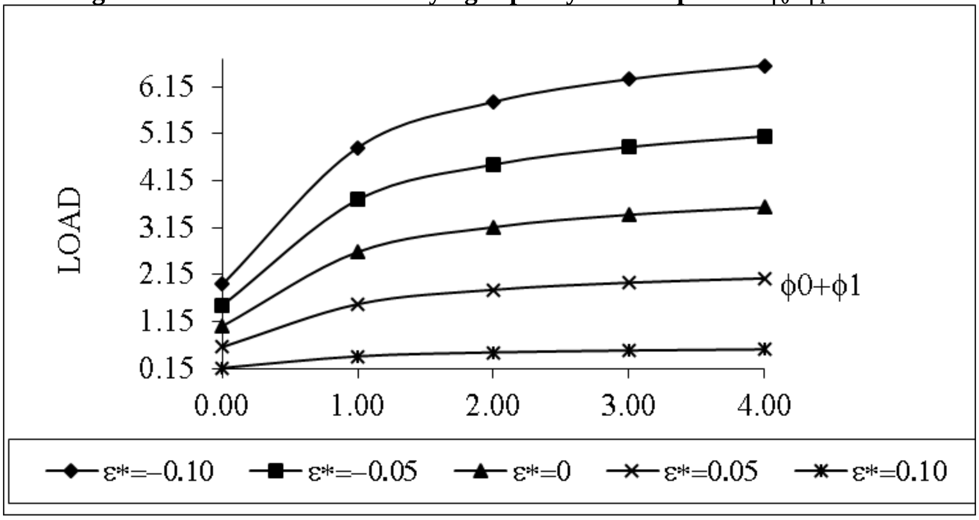

Figure 8: Distribution of load with respect to $\phi_0+\phi_1$ and ε*

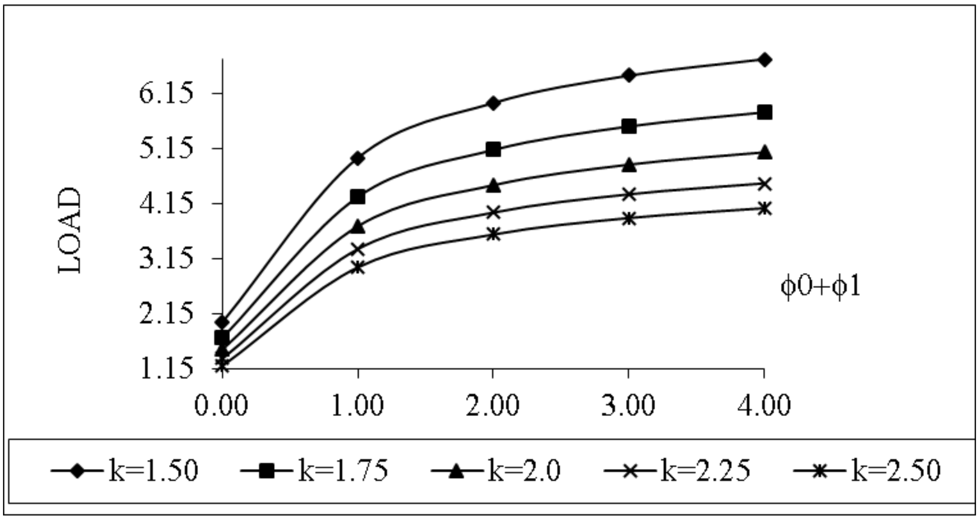

Figure 9: Profile of load bearing capacity with regards to $\phi_0+\phi_1$ and k

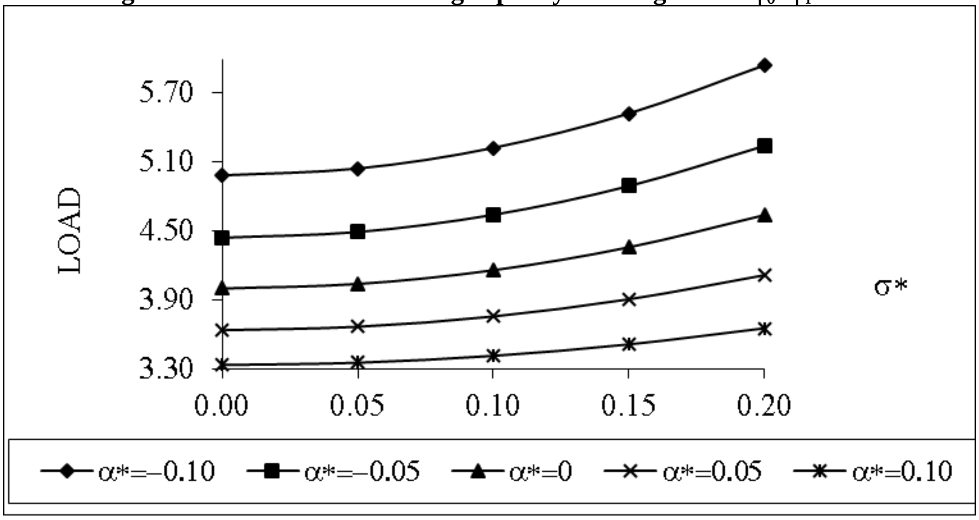

Figure 10: Variation of load carrying capacity with respect to σ* and α*

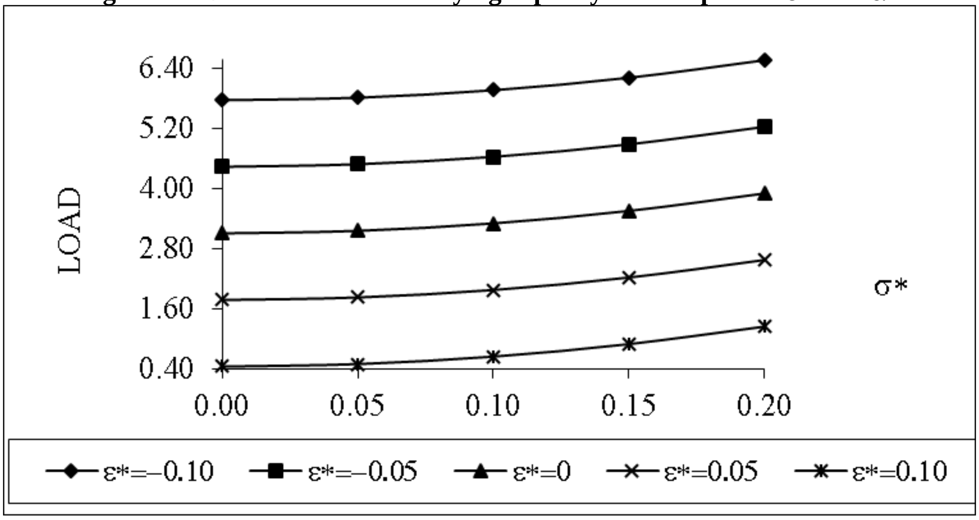

Figure 11: Distribution of load with respect to σ* and ε*

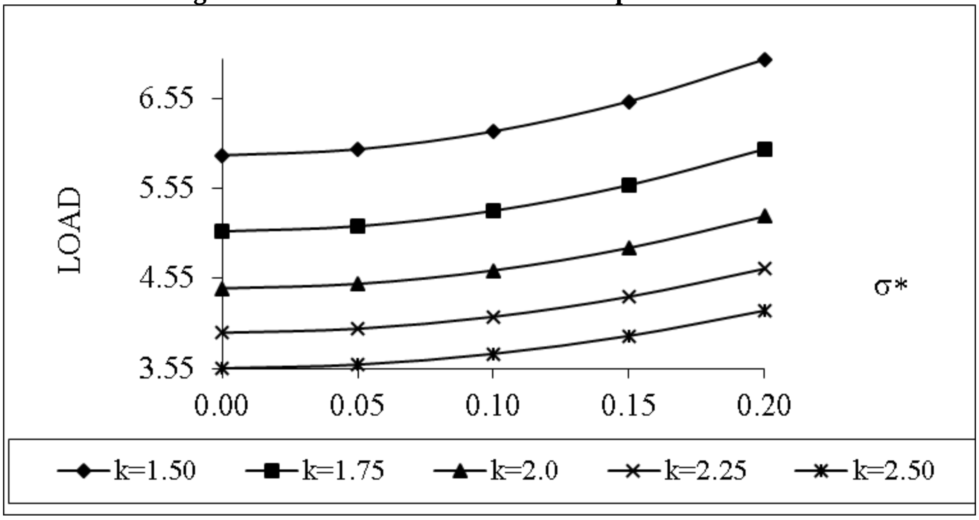

Figure 12: Profile of load bearing capacity with regards to σ* and k

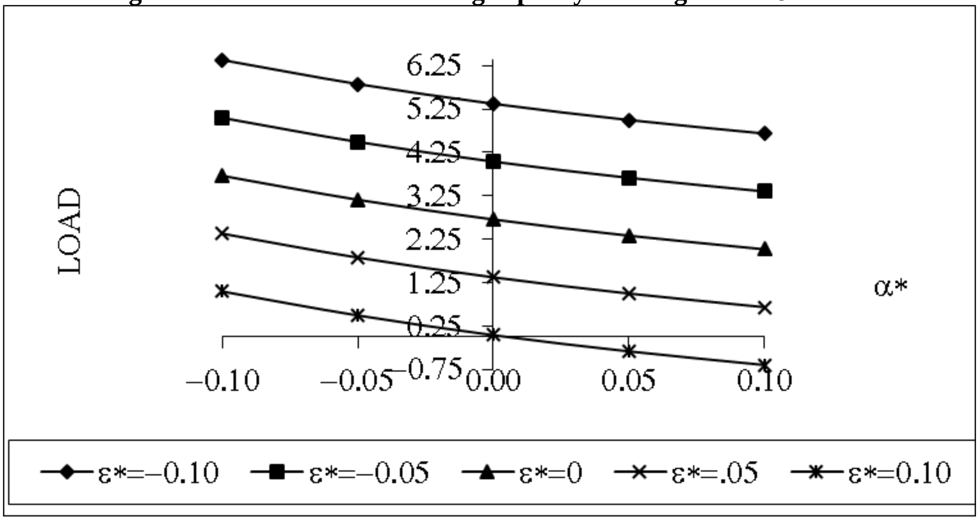

Figure 13: Variation of load carrying capacity with respect to α* and ε*

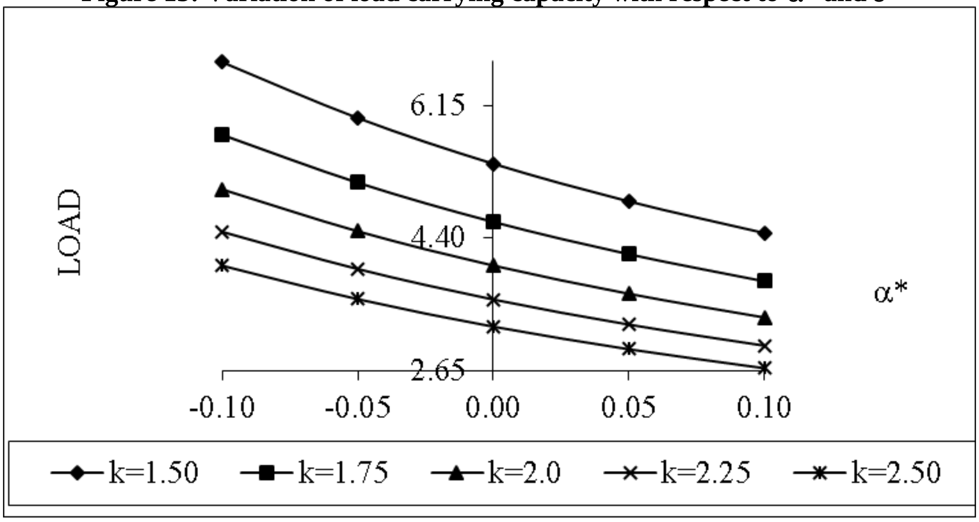

Figure 14: Distribution of load with respect to α* and k

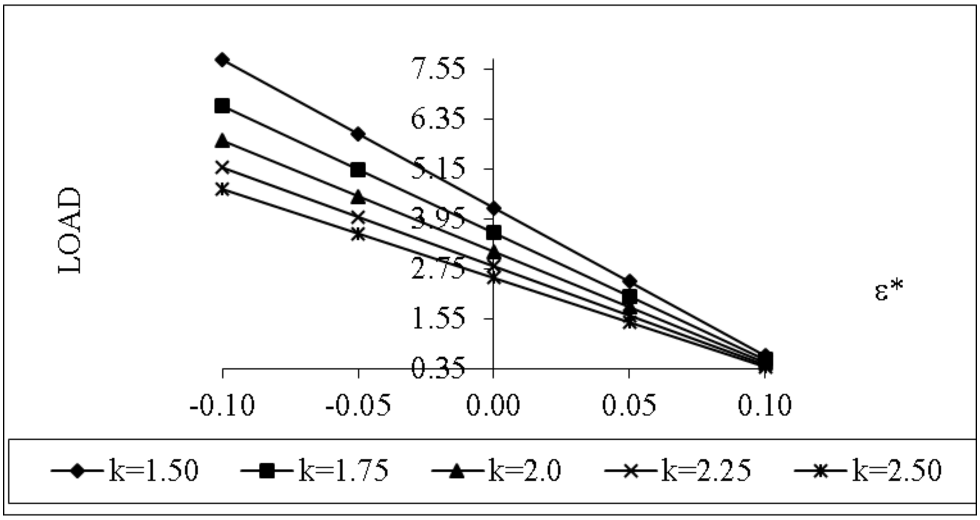

Figure 15: Profile of load bearing capacity with regards to ε* and k