Transversely rough circular plate model: Hip joint lubrication

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***Abstract*— In this article, the effect of surface roughness and boosted lubrication have been analyzed for the circular joint geometry. The synovial fluid has been taken to be a pseudo plastic fluid. The effects of cartilage porosity have not been considered. Here, the load bearing region of the joint has been approximated by two circular parallel plates which is more suited to the hip joint case. The stochastic model of Christensen and Tonder has been brought in to account for surface roughness. The associated stochastically averaged non-linear differential equation is solved with the consideration of appropriate boundary conditions to obtain the pressure distribution, in turn, which results in the calculation of load bearing capacity. The results presented in the graphical forms establish that the boosting effect combined with cartilage surface roughness decreases the load carrying capacity in general. However, the boosting parameter increases the load carrying capacity and enhances the performance by reducing the adverse effect of roughness.**

***Keywords*— Circular plates, Pressure, Hip joint lubrication, Roughness, Load carrying capacity.**

I. Introduction

The use of hydrodynamic lubrication theory is well known in the realm of hip joint replacements. Many models have been presented in this regard. However, most of the studies have assumed the constant surfaces to be smooth, which is in fact not true in reality, Indeed the surfaces are rough to certain extent. Therefore, it was thought proper to evaluate the effect of surface roughness on the hydrodynamic lubrication in the context of hip joint replacement.

A number of theoretical and experimental studies on the hydromagnetic lubrications for porous as well as plane metal bearings have appeared. The investigation of Elco and Huges [1] concerning the hydromagnetic lubrication of liquid metals bearing has attracted considerable attention with regards to the context of hip joint replacement.

Rao et. al. [2] presented a numerical scheme for hydrodynamic lubrication of synovial fluid. A critical Reynolds' number was found to be around 600 beyond which the load increased abnormally. Wingstrand and Wingstrand [3] developed a mathematical model to calculate forces, tension and stretching in the hip joint capsule under conditions caused by the joint effusion. This model was based on experimental data. It was established that the shape was distorted in a hip with effusion, rotation resulting in an increased intra capsular pressure and tension in the capsule with potential risk ruptures. Pietal [4] presented a modeling of human joints within the framework of micro polar fluid theory. Barbour et. al. [5] conducted a hip joint simulator study by making use of simplified loading and motion cycles generating physiological wear paths and rates. Norman et. al. [6] evaluated a collarless, tapered femoral total hip stem with an unsupported distal tip using a physiological three-dimensional finite element analysis. It was concluded that polished stem had a greater potential to develop taper lock fixation than do rough stem. Daniel [7] discussed the mathematical simulation of the hip joint loading. Bachtar et. al. [8] conducted a finite element contact analysis of the hip joint. In this study a contact smoothing approach was introduced by applying the Gregory patches. Williams et. al. [9] embarked on a theoretical modeling for lubrication using elastohydrodynamic theory, the results showed that the performance of metal-on-metal bearings was highly dependent on swing phase load. Therefore, the tension of the tissues at surgery could increase swing phase load, reduce lubrication, increase friction and accelerate wear.

Vadher et. al [10] studied the behaviour of hydromagnetic squeeze films between two conducting rough porous circular plates. This investigation indicated that the adverse effect of transverse roughness could be minimized to a large extent by the positive effect of magnetization when negatively skewed roughness occurred. Besides, this investigation confirmed that the bearing with magnetic field could support certain amount of load even when there was no flow, unlike the case of convectional fluid. Copper et. al. [11] established that the use of ceramic femoral heads in artificial hip joints helped to pressure the smooth surface finish on the femoral bearing surface thereby, ensuring low wear rates. Angadji et. al. [12] proved that large cusp inclination angles not only moved deposition of the wear scar but also increased the wear rates and total wear volume generated. Mattel et al. [13] reviewed the progress in lubrication and wear modeling of artificial hip joints. It was pointed out that actually, lubrication and wear were described neglecting each other while some advanced models included both the aspects. Srimongkal [14] reviewed the mathematical modeling analysis in total hip replacements. This review was focused on heat transfer, fluid flow and stress distribution models. Nemade and Tripathi [15] discussed a mathematical model for calculating contact stresses in artificial human hip joint. It was found that the radius of the femoral head had a significant influence on the peak contact stresses in the accelerated cavity.

II. ANALYSIS

The physical configuration of the hip joint represented by two rough circular plates is shown in Figure (1).

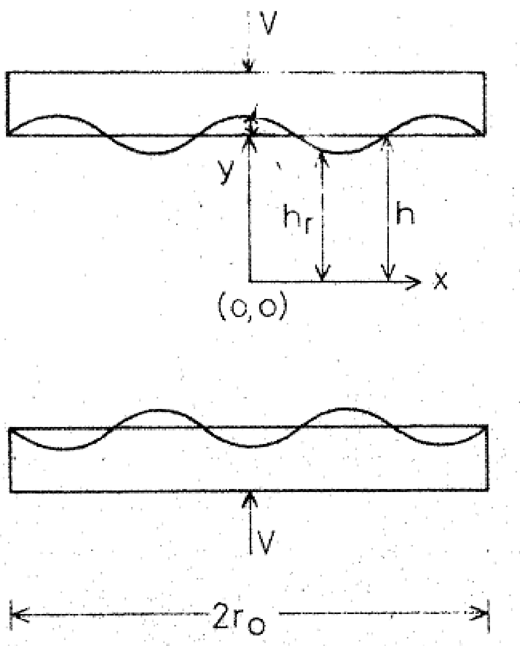


FIGURE: 1 CONFIGURATION OF THE BEARING SYSTEM.

Following the discussion of Peeyush Chandra [16] the basic equation governing the pressure distribution for smooth surfaces in this case turns out to be

$$\frac{1}{r}\frac{d}{dr}\left(\frac{h_r^{2+\frac{1}{n}}}{2+\frac{1}{n}}.r\left(-\frac{1}{m}\frac{dp}{dr}\right)^{\frac{1}{n}}\right)=V$$

… (1)

It is well known by now that the roughness of the bearing surfaces has a significant impact on the performance characteristics of the bearing system. Sometimes, even the contamination of the lubricant and chemical degradation of the surfaces contribute to roughness. Following the studies of Christensen and Tonder [17, 18, 19] the film thickness h(x) can be described as

$$h(x) = \bar{h}(x) + hs(x)$$

… (2)

where $\bar{h}(x)$ is the nominal film thickness between the mean level of the bearing surfaces and hs(x) is the random deviation from the mean film thickness being governed by a probability density function f (hs), where

$$f\left(h_s\right)=\begin{cases}\frac{35}{32C^7}(C^2-h_s^2)^{3/2}; -C\le h_s\le C\\ \\ 0;\ \text{elsewhere}\end{cases}$$

… (3)

'C' being the maximum deviation in 'h'. The details regarding the standard deviation, measure of symmetry and mean can be had from the above study of Christensen and Tonder [17, 18, 19]. In view of the stochastic modeling of Christensen and Tonder [17, 18, 19] the above equation (1) transforms to

$$\frac{1}{r}\frac{d}{dr}\left(\frac{g(h_r)^{\frac{2}{3}+\frac{1}{3n}}}{\frac{2}{3}+\frac{1}{3n}}.r\left(-\frac{1}{m}\frac{dp}{dr}\right)^{\frac{1}{n}}\right)=V$$

… (4)

where

$$g(h_r) = h_r^3 + 3\sigma^2 h_r + 3\alpha h_r^2 + 3\alpha^2 h_r + 3\sigma^2\alpha + \alpha^3 + \varepsilon$$

The associated boundary conditions are

p(r0) = 0; = 0 at r = 0

… (5)

Now, integrating equation (4) with respect to boundary conditions (5), one obtains the expression for pressure distribution as,

$$p = m\left(\frac{2n+1}{6n}V\right)^n \frac{1}{g\left(h_r\right)^{\frac{2}{3n}+\frac{1}{3}}}\cdot\frac{1}{(n+1)}\left(r_0^{n+1}-r^{n+1}\right)$$

… (6)

Then the load carrying capacity in this case, defined by

$$w = 2\pi\int_0^{r_0} p(r)\cdot rdr$$

leads to

$$w = \frac{\pi m_0}{n+3}\left(\frac{2n+1}{6n}V\right)^n \frac{r_0^{\;n+3}}{h_r^{\;2n+1}\, g\left(h_r\right)^{\frac{2}{3n}+\frac{1}{3}}} \cdot \frac{1}{(n+1)}\left(1+\frac{G}{h_r}\right)$$

… (7)

The introduction of the following dimensionless quantities

$\sigma* = \sigma/\eta$, $\varepsilon* = \varepsilon/\eta 3$, $\alpha* = \alpha/\eta$,

$$W = -\ \frac{wh^3}{\mu \dot{h}\, \pi^2 r^4}$$

gives dimensionless load carrying capacity as

$$W = \frac{\left(\frac{1}{3}\right)^n \left(\frac{h}{h_r}\right)^{2n+1}}{g\left(\bar{h}_r\right)^{\frac{2n+1}{3}}} \cdot \left(1+\frac{G}{h_r}\right)$$

… (8)

III. RESULTS AND DISCUSSION

It is seen that the load carrying capacity is determined by equation - (8). It is clearly seen that in the absence of boosting the roughness turns in very significant effect on the performance of the bearing system. Besides, the ratio of mean fluid film thickness to the thickness of rough surfaces has a crucial effect. Further, in the absence of roughness the role of the above ratio becomes more predominant. Besides, the load carrying capacity increases when the ratio of boosting parameter to the film thickness for rough surface increases because the expression involved in the equation is linear with respect to the above parameter G/hr.

The profiles of load carrying capacity with respect to the ratio G/hr presented in Figures (2) – (5) indicates that the load carrying capacity increases almost sharply due to the boosting.

Figure: 2 Variation of load carrying capacity with respect to G/hr and n.

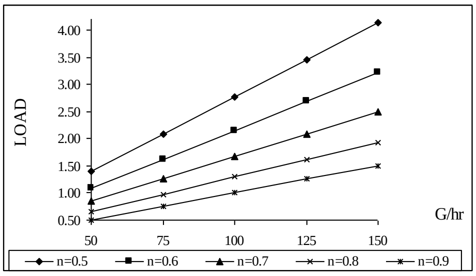


Figure: 3 Distribution of load carrying capacity with respect to G/hr and h/hr.

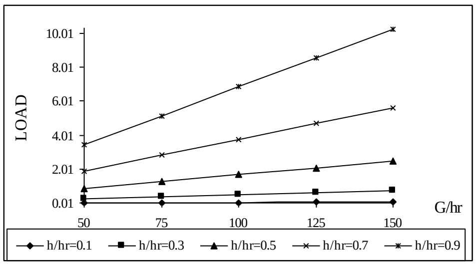


Figure: 4 Profile of load carrying capacity with respect to G/hr and σ∗.

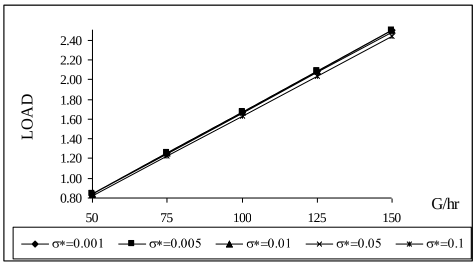
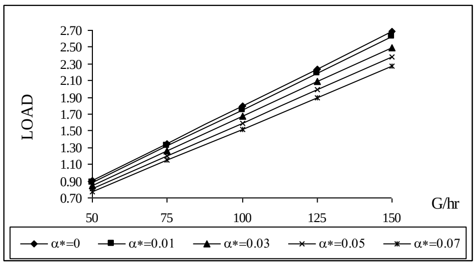


Figure: 5 Distribution of load carrying capacity with respect to G/hr and α∗.



However, the effect of standard deviation on the load carrying capacity with respect to G/hr remains almost negligible as can be seen from Figure (4). The effect of flow index (n) found in Figures (6)–(10) suggest that there is a heavy load reduction due to the flow index. But, here also the standard deviation introduces a negligible effect Figure (8).

Figure: 6 Profile of load carrying capacity with respect to G/hr and ε∗.

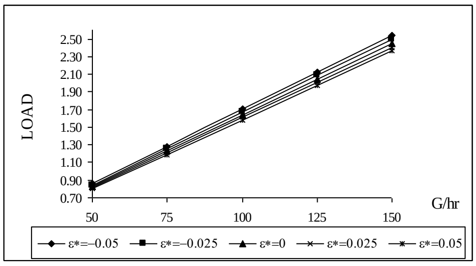


Figure: 7 Variation of load carrying capacity with respect to n and h/hr.

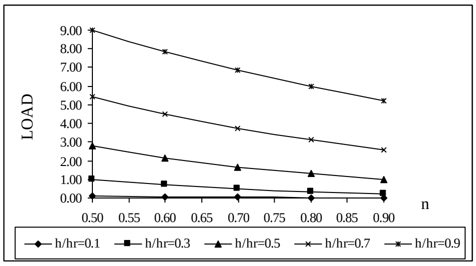


Figure: 8 Distribution of load carrying capacity with respect to n and σ∗.

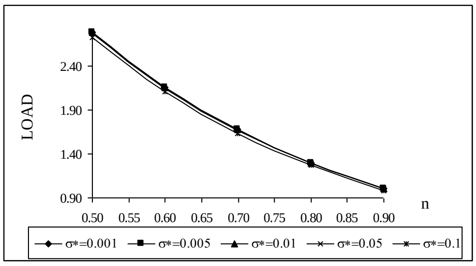


Figure: 9 Variation of load carrying capacity with respect to n and α∗.

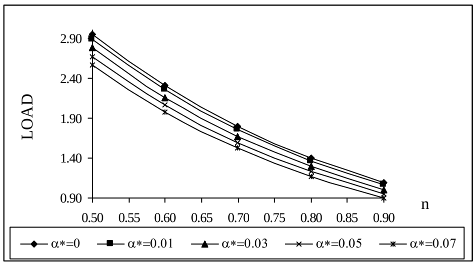
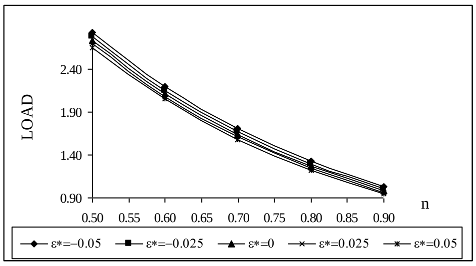


Figure: 10 Profile of load carrying capacity with respect to n and ε∗.



The combined effect of standard deviation and skewness on the load carrying capacity with respect to the ratio h/hr is at the best negligible which can be seen from Figures (11) and Figures (13).

Figure: 11 Variation of load carrying capacity with respect to h/hr and σ∗.

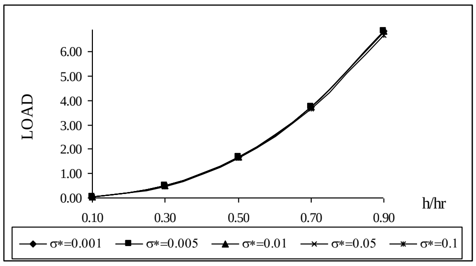
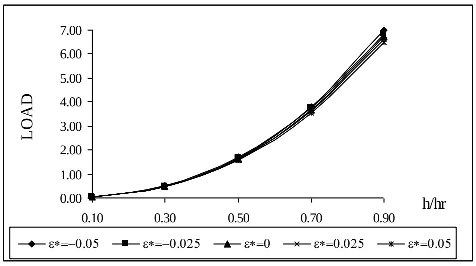
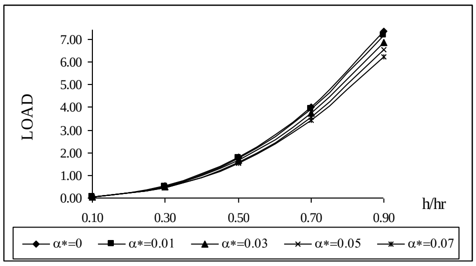


Figure: 13 Distribution of load carrying capacity with respect to h/hr and ε∗.



Even the effect of variance remains negligible for thickness ratio less than 0.4 (Figure (12)). Of course, the load increases with increasing values of h/hr.

Figure: 12 Variation of load carrying capacity with respect to h/hr and α∗.



The effect of standard deviation is to reduce the load carrying capacity which can be viewed from Figures (14) – (15).

Figure: 14 Variation of load carrying capacity with respect to σ∗ and α∗.

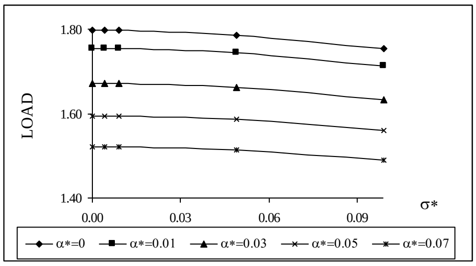
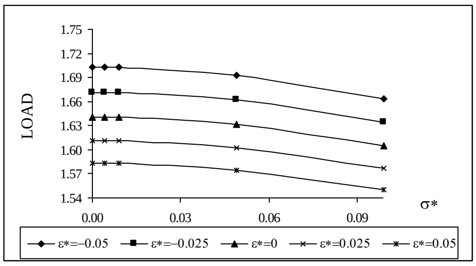
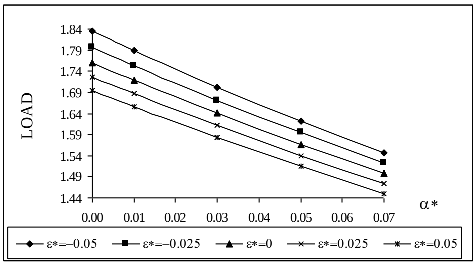


Figure: 15 Profile of load carrying capacity with respect to σ∗ and ε∗.



Lastly, Figure (16) underlines that the variance affects the system adversely by reducing the load.

Figure: 16 Variation of load carrying capacity with respect to α∗ and ε∗.



However, the negatively skewed roughness turns in increased pressure. It is observed that the results presented here compare well with the already known results.

IV. CONCLUSION

1. This investigation conclusively establishes that with proper boosting the negatively skewed roughness may present a better opportunity.
2. The roughness aspects must be evaluated carefully.
3. If properly designed this investigation may be helpful for hip joint replacements.

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