Squeeze Film Performance between a Rectangular Plate and a Rough Porous Surface

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Abstract— **This investigation aims to analyze a squeeze film performance in between a rectangular plate and a rough porous surface. The roughness has been characterized by the stochastic model of Christensen and Tonder. Two different forms of the probability distribution functions have been discussed. The associated stochastically averaged Reynolds' type equation has been solved to get the pressure distribution; afterword the load carrying capacity is calculated. The results in graphical form conform that although the porosity effect remains negligible up to certain extend, the effect of transverse surface roughness remains adverse but the situation is a little better in the case of negatively skewed roughness.**

***Keywords*—** **Squeeze film; Rectangular plates; Porosity; Roughness; Load carrying Capacity.**

I. Introduction

Wu [9] analyzed the squeeze film performance when one of the surfaces was porous faced for mainly, two types of geometries namely, annular and rectangular. Prakash and Vij [10] observed the behavior of squeeze film taking several geometries of the bearing surfaces. Here also, rectangular geometry was considered.

By now it is a well-known fact that the transverse roughness of the bearing surfaces affects the bearing performance adversely. This makes inevitable to analyze the effect of transverse roughness on the squeeze film behavior, which is essential from bearing life period point of view.

The effect of surface roughness was discussed by several investigators (Tonder [17], Tzeng and Saibel [14], Christensen and Tonder [2, 3, 16]). The model developed by Christensen and Tonder [2, 3, 16] played a key role in investigating the effect of surface roughness. (Prajapati [6, 19], Andharia et. al. [11], Patel et .al. [18], Siddangouda et. al. [4]). Deheri et. al. [7] found that the negatively skewed roughness provided a better chance to improve the bearing performance. Patel et. al.[8] discussed the effect of surface roughness with ferrofluid lubrication on the truncated conical plates with variable boundary conditions. A comparative study of different roughness structures had been discussed by Acharya et. al.[5], which confirms that the trigonometric form is more favorable then the algebraic structure. Majumdar [1] studied the performance of thickness ratio on rectangular plates on a plane surface. It was observed that the thickness ratio had a profound impact on the bearing performance. The hydromagnetic squeeze film between porous rectangular plates was studied by Fatima et.al. [15]. Kudenatti et al. [13] analyzed the effects of surface roughness and couple-stress fluid between two rectangular plates using the MHD Reynolds equation for Squeeze-Film lubrication.

Kudenatti et. al. [12] studied the characteristic features of squeeze film lubrication between two rectangular plates, upper plate having rough, in the presence of a uniform transverse magnetic field. The associated Reynolds' equation was solved by the help of a multi grid method. The load carrying capacity and squeeze film time were found to increase for small values of couple stress parameter. Further, the increased roughness parameter caused increased load carrying capacity.

In all the above studies the roughness influences the bearing performance significantly. Therefore, it was thought appropriate to examine the effect of transverse roughness on the squeeze film performance between a rectangular plate and a rough porous surface considering two different forms of the roughness patterns.

II. Analysis

The configuration of the bearing system is presented below.

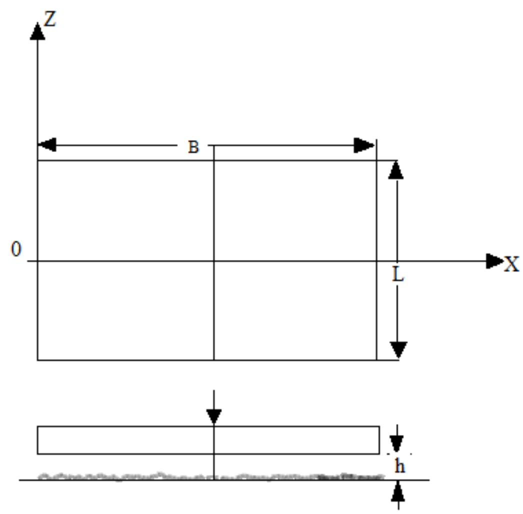

Figure- I Configuration of the bearing

The lower surface is fixed having porous face. The upper rectangular plate moves towards the lower surface along the normal with a velocity $\dot{h} = \frac{dh}{dt}$. The clearance space between the plates and the porous surface is filled by a lubricant.

The transverse surface roughness of the lower surface is characterized by a random variable with non-zero mean, variance and skewness. The transverse surface roughness of the bearing surface is characterized by a random variable with non-zero mean, variance and skewness. Following the discussions of Christensen and Tonder [8, 9, 10], the film thickness h(x) is considered as

$$h(x) = \bar{h}(x) + h_s(x)$$

where $\bar{h}(x)$ is the mean film thickness and $h_s(x)$ is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. $h_s(x)$ is described by a probability density function $f(h_s)$,defined by

$$f(h_s) = \begin{cases} \frac{35}{32C^7}(C^2 - h_s^2)^3, if - C \le h_s \le C \\ 0, \quad eleswhere \end{cases}$$

C is being the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε, which is the measure of symmetry, of random variable h_s, are defined by the relationships

$$\alpha = E(h_s)$$

$$\sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\epsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by

$$E(R) = \int_{-C}^{C} R.\ F(h_s)\, dh_s$$

The details regarding the roughness and characterization can be seen from Christensen and Tonder [2, 3, 16].

In the light of the discussion of Christensen and Tonder [2, 3, 16] the modified Reynolds' equation associated with this system turns out to be

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = -\frac{12\mu\dot{h}}{g_i(h)} \text{ for } i = 1,2 \quad \text{..... (1)}$$

Where

$$g_1(h) = h^3 + 3\sigma^2 h + 3\alpha^2 h + 3h^2\alpha + 3\alpha\sigma^2 + \alpha^3 + \varepsilon + 12\emptyset H \quad \text{..... (2)}$$

Besides the second roughness pattern and discussion from Prajapati [19] lands us in

$$g_2(h) = h^3 + 4\sigma^2 h + 3\alpha^2 h + 2h^2\alpha + 4\alpha\sigma^2 + \alpha^3 + \varepsilon + 12\emptyset H \quad \text{..... (3)}$$

Assume that the solution of equation (1) can be assumed as

$$p(x,z) = \left(\sum_{n=1,3,5,..}^{\infty} C_n \cos\frac{n\pi x}{B}\right) Z(z) \quad \text{..... (4)}$$

Where $Z(z)$ an unknown function of z only and C_n is a constant.

The solution of this equation (1) in view of the boundary conditions

$$p = 0 \text{ at } z = \pm\frac{L}{2}$$

derives the pressure distribution as

$$p(x,z) = \sum_{n=1,3,5,..}^{\infty} C_n \cos\frac{n\pi x}{B}\left(-\frac{48\mu\dot{h}B^2}{C_n\pi^3\, g_i(h)}\,\frac{(-1)^{\frac{n-1}{2}}}{n^3}\,\frac{\cosh\frac{n\pi z}{B}}{\cosh\frac{n\pi L}{2B}} + \frac{48\mu\dot{h}B^2}{C_n\pi^3\, g_i(h)}\,\frac{(-1)^{\frac{n-1}{2}}}{n^3}\right)$$

$$= -\frac{48\mu\dot{h}B^2}{\pi^3\, g_i(h)}\,\frac{(-1)^{\frac{n-1}{2}}}{n^3}\sum_{n=1,3,5,..}^{\infty}\frac{(-1)^{\frac{n-1}{2}}}{n^3}\left(\frac{\cosh\frac{n\pi z}{B}}{\cosh\frac{n\pi L}{2B}} - 1\right)\cos\frac{n\pi x}{B} \qquad \text{..... (5)}$$

The squeeze load capacity W is computed as

$$W = 2\int_0^{\frac{L}{2}}\int_0^{B} p(x,z)\,dx\,dz$$

$$=2\int_0^{\frac{L}{2}}\int_0^{B}\left\{-\frac{48\mu\dot{h}B^2}{\pi^3\, g_i(h)}\,\frac{(-1)^{\frac{n-1}{2}}}{n^3}\sum_{n=1,3,5,..}^{\infty}\frac{(-1)^{\frac{n-1}{2}}}{n^3}\left(\frac{\cosh\frac{n\pi z}{B}}{\cosh\frac{n\pi L}{2B}} - 1\right)\cos\frac{n\pi x}{B}\right\}dx\,dz$$

$$= \frac{192\mu\dot{h}B^4}{\pi^4\, g_i(h)}\sum_{n=1,3,5,..}^{\infty}\left(\frac{L}{2Bn^4} - \frac{1}{\pi\, n^5}\tanh\frac{n\pi L}{2B}\right) \qquad \text{..... (6)}$$

The dimensionless form of W is

$$\bar{W} = \frac{h^3}{\mu\dot{h}L^4}W$$

$$= \frac{192}{\pi^4 G_i}\left(\frac{B}{L}\right)^4\sum_{n=1,3,5,..}^{\infty}\left(\frac{L}{2Bn^4} - \frac{1}{\pi\, n^5}\tanh\frac{n\pi L}{2B}\right) \text{ for } \quad i = 1,2 \qquad \text{..... (7)}$$

where

$$G_1 = 1 + 3\sigma^{*2} + 3\alpha^{*2} + 3\alpha^{*} + 3\alpha^{*}\sigma^{*2} + \alpha^{*3} + 12\Psi \qquad \text{..... (8)}$$

and

$$G_2 = 1 + 4\sigma^{*2} + 3\alpha^{*2} + 2\alpha^{*} + 4\alpha^{*}\sigma^{*2} + \alpha^{*3} + 12\Psi \qquad \text{..... (9)}$$

III. Results and Discussions

Equation (7) represents the expression of load carrying capacity. In the absence of roughness this study reduces to the investigation of Majumdar [1].To analyze the effect of various roughness parameters, porosity and aspect ratio the following graphical representation are presented.

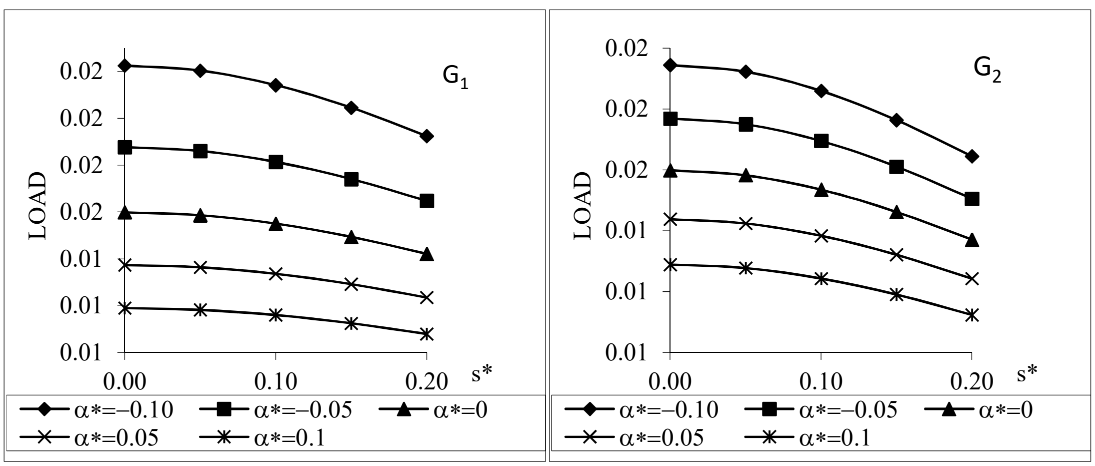

Fig. 1 Variation of load carrying capacity with respect to σ* and α*.

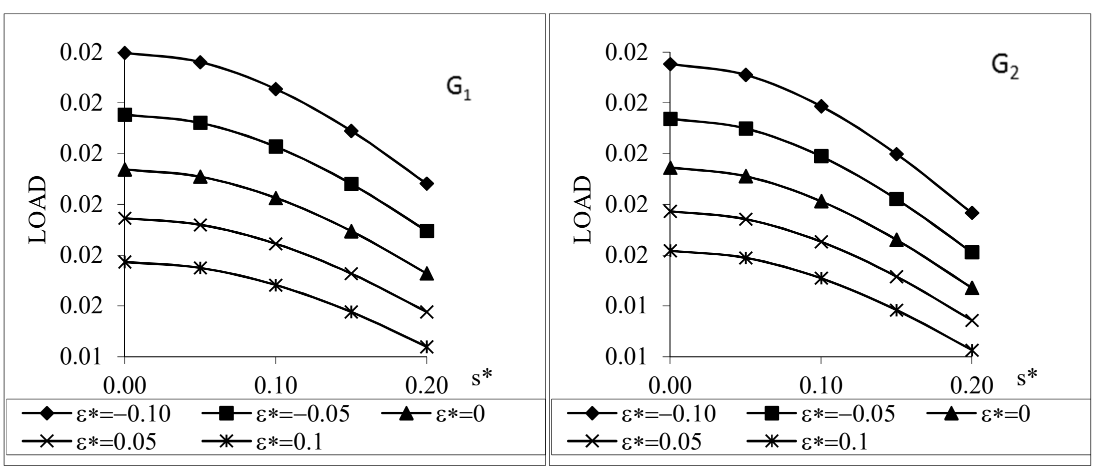

Fig.2 Variation of load carrying capacity with respect to σ* and ε*.

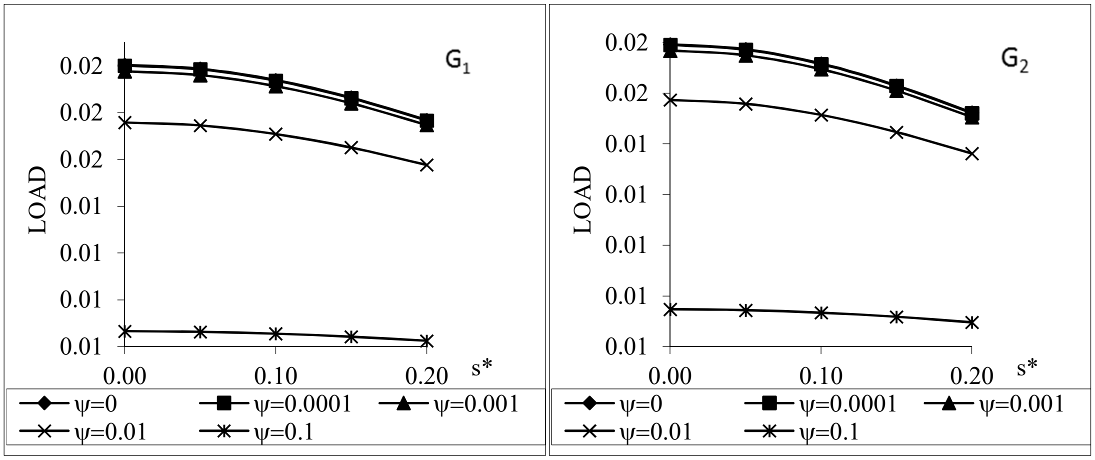

Figure: 3 Variation of load carrying capacity with respect to σ* and ψ.

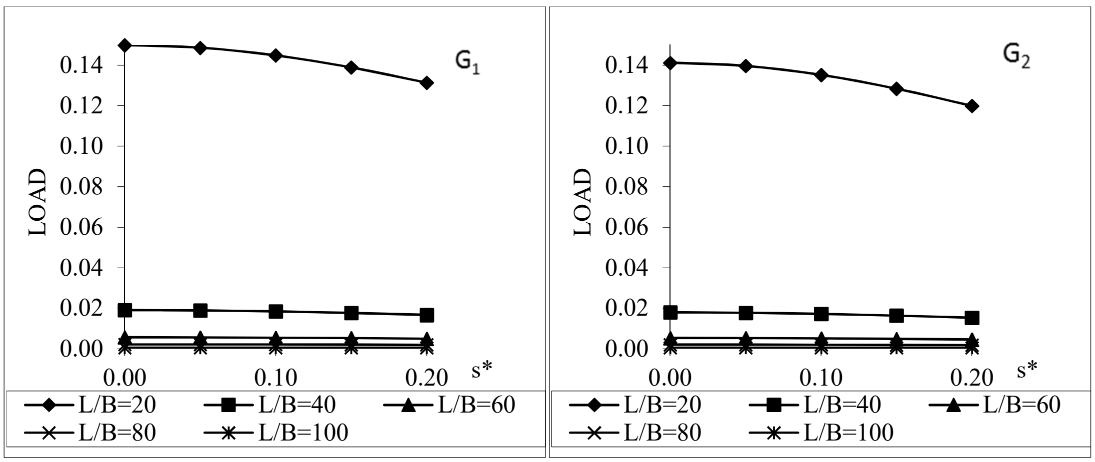

Figure: 4 Variation of load carrying capacity with respect to σ* and L/B.

The fact that standard deviation brings down the load bearing capacity can be seen from Figures (1) – (4) for the both forms of roughness.

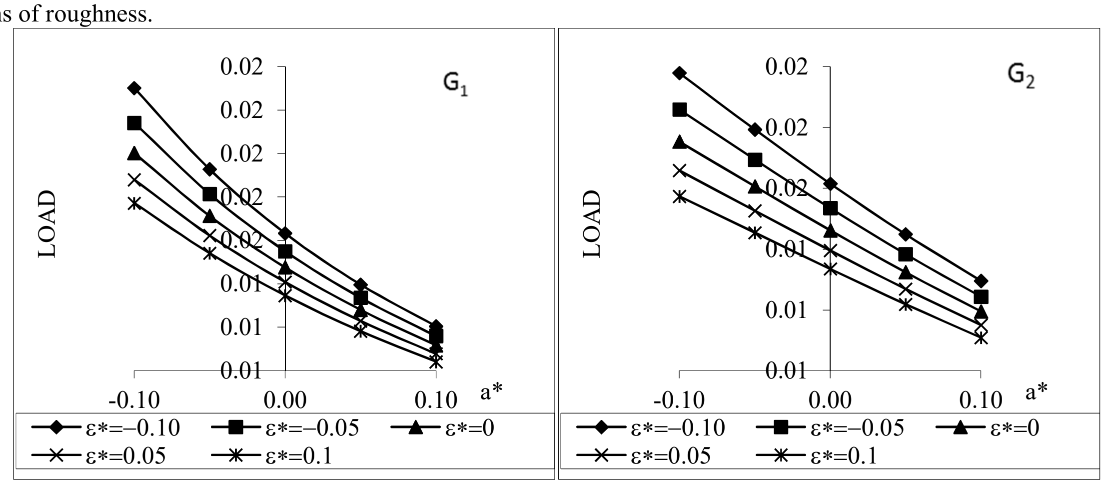

Figure: 5 Variation of load carrying capacity with respect to α* and ε*.

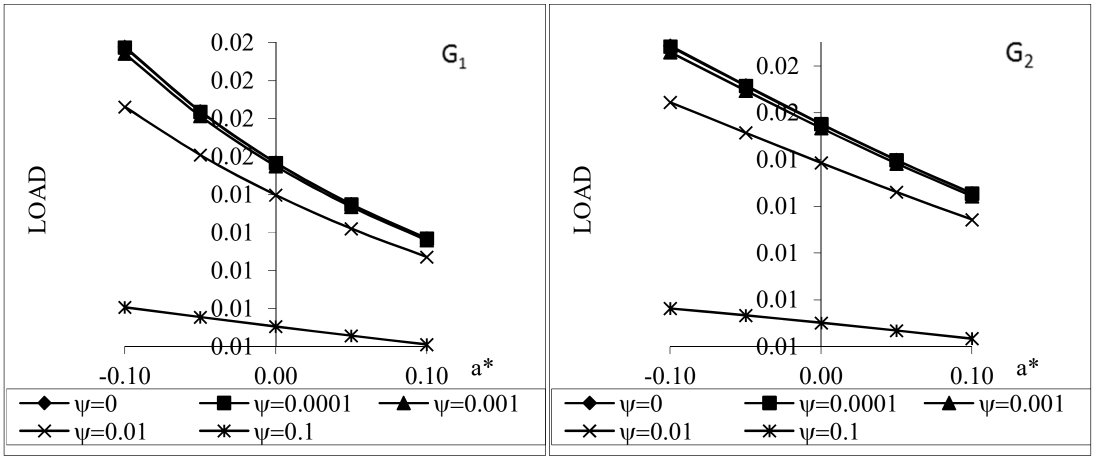

Figure: 6 Variation of load carrying capacity with respect to α∗ and ψ.

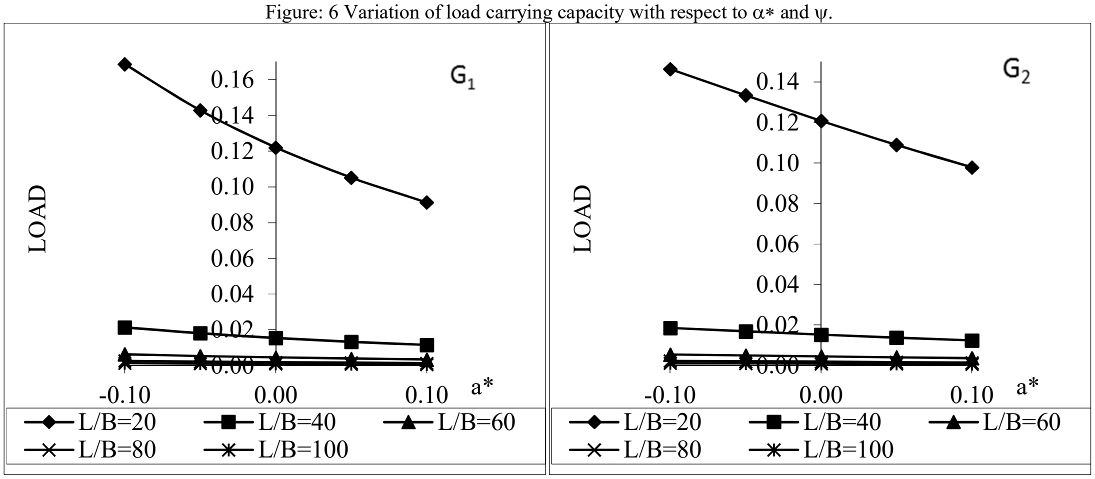

Figure: 7 Variation of load carrying capacity with respect to α∗ and L/B.

The performance remains relatively better while the negatively skewed roughness occurs, which can be seen from Figures (5) to (7).

The trends of load with respect to variance are almost identical with that of skweness. Therefore, the bearing tends to register a better performance with the involvement of negatively skewed roughness. This performance further moves up with the advent of variance (-ve).

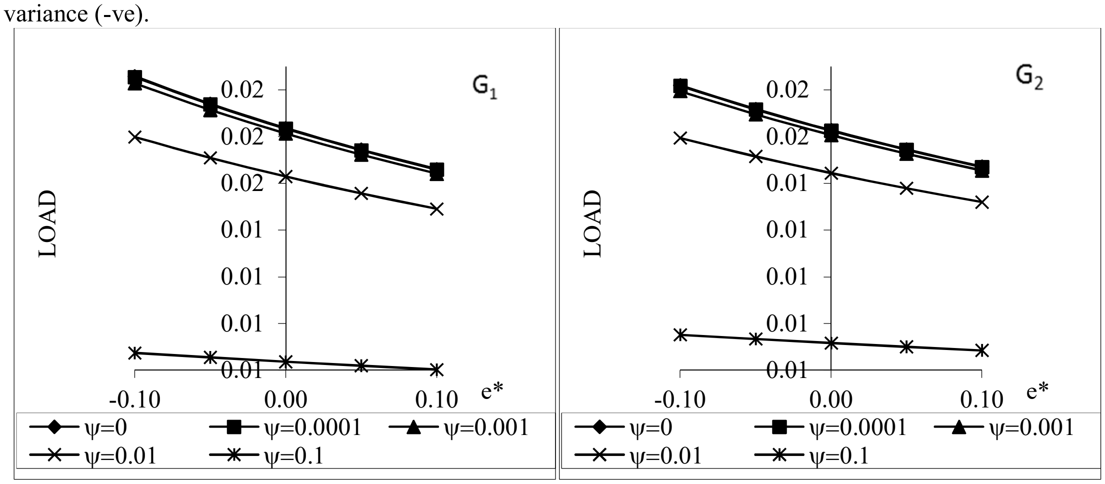

Figure: 8 Variation of load carrying capacity with respect to ε∗ and ψ.

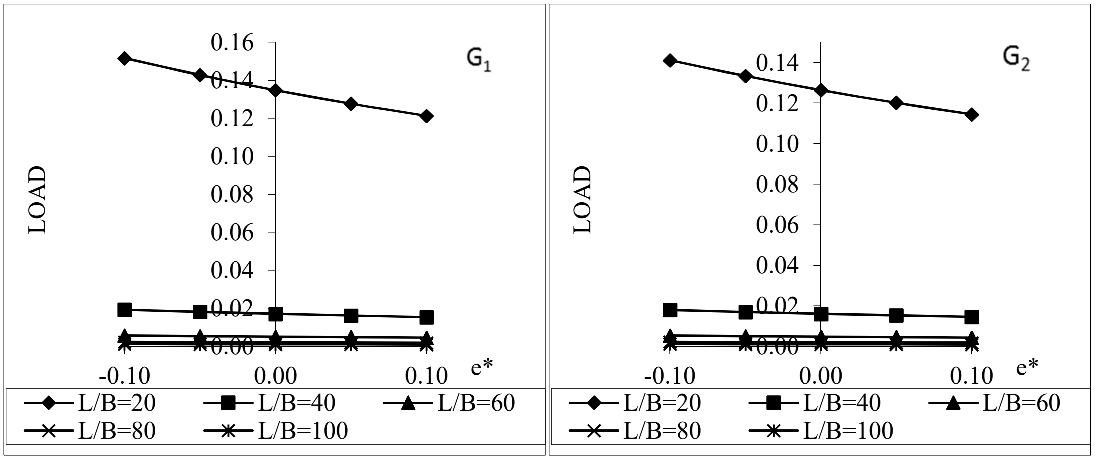

Figure: 9 Variation of load carrying capacity with respect to ε* and L/B.

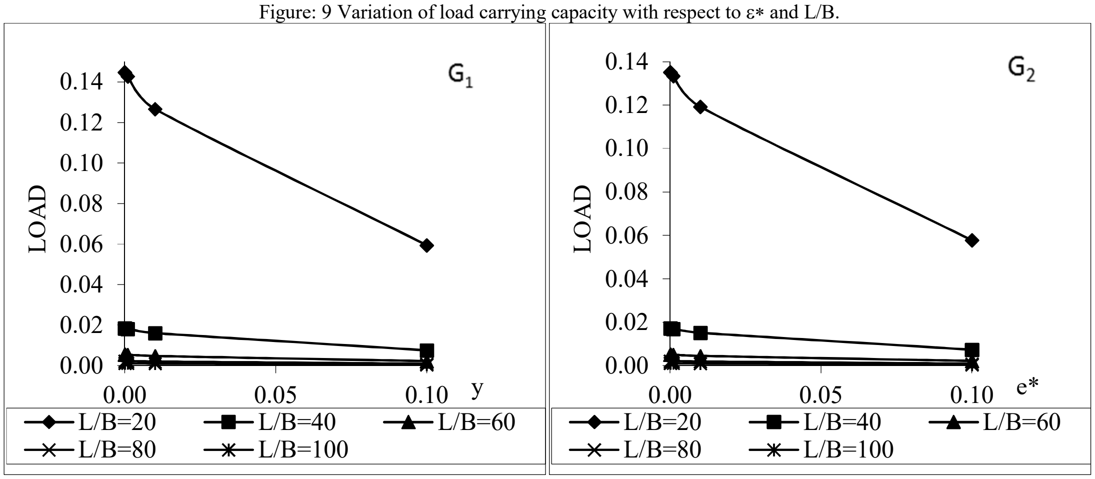

Figure: 10 Variation of load carrying capacity with respect to ψ and L/B.

Further it is appealing to note that the porosity effect remains almost nominal up to the porosity value 0.001. However, the graphical comparison indicates that the first characterization remains favourable respect to second.

IV. Conclusion:

It is observed that the porosity effect remains negligible up to the value 0.001 approximately. However, as usual it decreases load taking capacity sharply for greater values of porosity. Out of three roughness parameters skewness affects the most, as it helps the bearing to support the load in case of negative skewness. Side by side, the influence of variance is equally important, which increase in the case of variance (-ve). Further, the second form of roughness may be given due consideration according to the geometry of the bearing system.

V. Nomenclature

h	Fluid film thickness
H	Thickness of the porous facing
p	Pressure
σ	Standard deviation
α	Variance
ε	Skewness parameter
ϕ	Permeability of the porous facing
Ψ	Porosity parameter($\phi H/h^3$)
B	Width of the rectangular plate
L	Length of rectangular plate
σ^*	Non-dimensional standard deviation(σ/h)
α^*	Non-dimensional variance (α/h)
ε^*	Non-dimensional skewness(ε/h^3)
W	Squeeze load capacity
$\bar{W}$	Non-dimensional Squeeze load capacity
μ	Absolute viscosity of lubricant

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